

Mark Scheme (Results)

January 2008

GCE

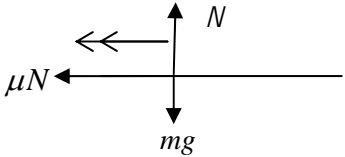
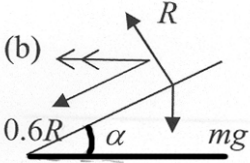
GCE Mathematics (6679/01)

January 2008
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.(a)	$T \text{ or } \frac{\lambda \times e}{l} = mg \quad (\text{even } T=m \text{ is M1, A0, A0 sp case})$ $\frac{\lambda \times 0.16}{0.4} = 2g$ $\Rightarrow \lambda = 49 \text{ N} \quad \text{or } 5g$	<p>M1</p> <p>A1</p> <p>A1 (3)</p>
1.(b)	<div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>Special case $T \sin \theta = mg$ giving $\theta = 30$ is M1 A0 A0 unless there is evidence that they think θ is with horizontal – then M1 A1 A0</p> </div> $R(\uparrow) \quad T \cos \theta = mg \text{ or } \cos \theta = \frac{mg}{T}$ $49 \cdot \frac{0.32}{0.4} \cdot \cos \theta = 19.6 \text{ or } 4g \cdot \cos \theta = 2g \text{ or } 2mg \cdot \cos \theta = mg \quad (\text{ft on their } \lambda)$ $\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \quad (\text{or } \frac{\pi}{3} \text{ radians})$	<p>M1</p> <p>A1ft</p> <p>A1 (3)</p> <p>6</p>
2.	<p>a)</p> $m 'a' = \pm \frac{16}{5x^2}, \text{ with acceleration in any form (e.g. } \frac{d^2x}{dt^2}, v \frac{dv}{dx}, \frac{dv}{dt} \text{ or)}$ <p>Uses $a = v \frac{dv}{dx}$ to obtain $k v \frac{dv}{dx} = \pm k' \frac{32}{x^2}$</p> <p>Separates variables, $k \int v dv = k' \int \frac{32}{x^2} dx$</p> <p>Obtains $\frac{1}{2} v^2 = \mp \frac{32}{x} (+ C)$ or equivalent e.g. $\frac{0.1}{2} v^2 = - \frac{16}{5x} (+ C)$</p> <p>Substituting $x = 2$ if + used earlier or -2 if – used in d.e. $x = 2, v = \pm 8 \Rightarrow 32 = -16 + C \Rightarrow C = 48$ (or value appropriate to their correct equation)</p> $v = 0 \Rightarrow \frac{32}{x} = 48 \Rightarrow x = \frac{2}{3} \text{ m} \quad (\text{N.B. } -\frac{2}{3} \text{ is not acceptable for final answer})$	<p>B1</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 cao</p> <p>8</p>
	<p>N.B. $\frac{d}{dx} (\frac{1}{2} m v^2) = \frac{16}{5x^2}$, is also a valid approach.</p> <p>Last two method marks are independent of earlier marks and of each other</p>	

Question Number	Scheme	Marks																									
3.(a)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20%;"></td> <td style="width: 20%; text-align: center;">Large cone</td> <td style="width: 20%; text-align: center;">small cone</td> <td style="width: 20%; text-align: center;">S</td> <td style="width: 20%;"></td> </tr> <tr> <td>Vol.</td> <td style="text-align: center;">$\frac{1}{3}\pi(2r)^2(2h)$</td> <td style="text-align: center;">$\frac{1}{3}\pi r^2 h$</td> <td style="text-align: center;">$\frac{7}{3}\pi r^2 h$</td> <td style="text-align: center;">(accept ratios 8 : 1 : 7)</td> </tr> <tr> <td>C of M</td> <td style="text-align: center;">$\frac{1}{2}h$,</td> <td style="text-align: center;">$\frac{5}{4}h$</td> <td style="text-align: center;">\bar{x}</td> <td style="text-align: center;">(or equivalent)</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$\frac{8}{3}\pi r^2 h \cdot \frac{1}{2}h - \frac{1}{3}\pi r^2 h \cdot \frac{5}{4}h = \frac{7}{3}\pi r^2 h \cdot \bar{x}$</td> <td style="text-align: center;">or equivalent</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$\rightarrow \bar{x} = \frac{11}{28}h$</td> <td style="text-align: center;">*</td> </tr> </table>		Large cone	small cone	S		Vol.	$\frac{1}{3}\pi(2r)^2(2h)$	$\frac{1}{3}\pi r^2 h$	$\frac{7}{3}\pi r^2 h$	(accept ratios 8 : 1 : 7)	C of M	$\frac{1}{2}h$,	$\frac{5}{4}h$	\bar{x}	(or equivalent)		$\frac{8}{3}\pi r^2 h \cdot \frac{1}{2}h - \frac{1}{3}\pi r^2 h \cdot \frac{5}{4}h = \frac{7}{3}\pi r^2 h \cdot \bar{x}$			or equivalent		$\rightarrow \bar{x} = \frac{11}{28}h$			*	<p style="text-align: center;">B1</p> <p style="text-align: center;">B1, B1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1 (5)</p>
	Large cone	small cone	S																								
Vol.	$\frac{1}{3}\pi(2r)^2(2h)$	$\frac{1}{3}\pi r^2 h$	$\frac{7}{3}\pi r^2 h$	(accept ratios 8 : 1 : 7)																							
C of M	$\frac{1}{2}h$,	$\frac{5}{4}h$	\bar{x}	(or equivalent)																							
	$\frac{8}{3}\pi r^2 h \cdot \frac{1}{2}h - \frac{1}{3}\pi r^2 h \cdot \frac{5}{4}h = \frac{7}{3}\pi r^2 h \cdot \bar{x}$			or equivalent																							
	$\rightarrow \bar{x} = \frac{11}{28}h$			*																							
(b)	$\tan \theta = \frac{2r}{x} = \frac{2r}{\frac{11}{28}h}, = \frac{2r}{\frac{11}{14}r} = \frac{28}{11}$ <p style="text-align: center;">$\theta \approx 68.6^\circ$ or 1.20 radians</p> <p>(Special case – obtains complement by using $\tan \theta = \frac{2r}{x}$ giving 21.4° or .374 radians M1A0A0)</p>	<p style="text-align: center;">M1, A1</p> <p style="text-align: center;">A1 (3) 8</p>																									
	<p>Centres of mass may be measured from another point (e.g. centre of small circle, or vertex) The Method mark will then require a complete method (Moments and subtraction) to give required value for \bar{x}). However B marks can be awarded for correct values if the candidate makes the working clear.</p>																										

4. (a)	<p>Energy equation with at least three terms, including K.E term</p> $\frac{1}{2}mV^2 + ..$ $+ .. \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{a^2}{16}, +mg \cdot \frac{1}{2}a \cdot \sin 30, = \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}$ $\Rightarrow V = \sqrt{\frac{ga}{2}}$	<p>M1</p> <p>A1, A1, A1</p> <p>dM1 A1 (6)</p>
(b)	<p>Using point where velocity is zero and point where string becomes slack:</p> $\frac{1}{2}mw^2 =$ $\frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}, -mg \cdot \frac{3a}{4} \cdot \sin 30$ $\Rightarrow w = \sqrt{\frac{3ag}{8}}$ <p>Alternative (using point of projection and point where string becomes slack):</p> $\frac{1}{2}mw^2 - \frac{1}{2}mV_1^2, = \frac{mga}{16} - \frac{mga}{8}$ $\text{So } w = \sqrt{\frac{3ag}{8}}$	<p>M1</p> <p>A1, A1</p> <p>A1 (4)</p> <p>M1,A1 A1</p> <p>A1</p> <p>10</p>
	<p>In part (a) DM1 requires EE, PE and KE to have been included in the energy equation.</p> <p>If sign errors lead to $V^2 = -\frac{ga}{2}$, the last two marks are M0 A0</p> <p>In parts (a) and (b) A marks need to have the correct signs</p> <p>In part (b) for M1 need one KE term in energy equation of at least 3 terms with distance $\frac{3a}{4}$ to indicate first method, and two KE terms in energy equation of at least 4 terms with distance $\frac{a}{4}$ to indicate second method.</p> <p>SHM approach in part (b). (Condone this method only if SHM is proved)</p> <p>Using $v^2 = \omega^2(a^2 - x^2)$ with $\omega^2 = \frac{2g}{a}$ and $x = \pm \frac{a}{4}$.</p> <p>Using 'a' = $\frac{a}{2}$ to give $w = \sqrt{\frac{3ag}{8}}$.</p>	<p>M1 A1 A1</p> <p>A1</p>

5.(a)	 $\frac{mv^2}{r} = \mu N, = \mu mg$ $\mu = \frac{v^2}{rg} = \frac{21^2}{75 \times 9.8} = 0.6 \quad *$	M1, A1 A1 (3)
(b)	 $R(\uparrow) R \cos \alpha, \mp 0.6R \sin \alpha = mg$ $\Rightarrow R \left(\frac{4}{5} - \frac{3}{5} \cdot \frac{3}{5} \right) = mg \Rightarrow R = \frac{25mg}{11}$	M1, A1, A1 A1 (4)
(c)	$R(\leftarrow) R \sin \alpha, \pm 0.6R \cos \alpha = \frac{mv^2}{r}$ $v \approx 32.5 \text{ m s}^{-1}$	M1, A1, A1 dM1 A1cao (5) 12
	<p>In part (b) M1 needs three terms of which one is mg If $\cos \alpha$ and $\sin \alpha$ are interchanged in equation this is awarded M1 A0 A1</p> <p>In part (c) M1 needs three terms of which one is $\frac{mv^2}{r}$ or $mr\omega^2$ If $\cos \alpha$ and $\sin \alpha$ are interchanged in equation this is also awarded M1 A0 A1</p> <p>If they resolve along the plane and perpendicular to the plane in part (b), then attempt at $R - mg \cos \alpha = \frac{mv^2}{r} \sin \alpha$, and $0.6R + mg \sin \alpha = \frac{mv^2}{r} \cos \alpha$ and attempt to eliminate v</p> <p>Two correct equations Correct work to solve simultaneous equations Answer</p> <p>In part (c) Substitute R into one of the equations Substitutes into a correct equation (earning accuracy marks in part (b)) Uses $R = \frac{25mg}{11}$ (or $\frac{25mg}{29}$) Obtain $v = 32.5$</p>	M1 A1 A1 A1 (4) M1 A1 A1 M1A1 (5)

6.(a)	Energy equation with two terms on RHS, $\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{5ga}{2} + mga \sin \theta$	M1, A1 A1 cso (3)
(b)	$\Rightarrow v^2 = \frac{ga}{2}(5 + 4 \sin \theta) \quad *$ R(\ string) $T - mg \sin \theta = \frac{mv^2}{a}$ (3 terms) $\Rightarrow T = \frac{mg}{2}(5 + 6 \sin \theta) \text{ o.e.}$	M1 A1 A1 (3)
(c)	$T = 0 \Rightarrow \sin \theta, = -\frac{5}{6}$ Has a solution, so string slack when $\alpha \approx 236(.4)^\circ$ or 4.13 radians	M1, A1 A1 (3)
(d)	At top of small circle, $\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{5ga}{2} - \frac{mga}{2}$ (M1 for energy equation with 3 terms) $\Rightarrow v^2 = \frac{3}{2}ga = 14.7a$ Resolving and using Force = $\frac{mv^2}{r}$, $T + mg = m \cdot \frac{\frac{3}{2}ga}{\frac{1}{2}a}$ (M1 needs three terms, but any v) $\Rightarrow T = 2mg$	M1 A1 A1 M1 A1 A1 (6) 15
	Use of $v^2 = u^2 + 2gh$ is M0 in part (a)	

7.(a)	<p>(Measuring x from E) $2\ddot{x} = 2g - 98(x + 0.2)$, and so $\ddot{x} = -49x$</p> <p>SHM period with $\omega^2 = 49$ so $T = \frac{2\pi}{7}$</p>	M1 A1, A1 d M1 A1cso (5)
(b)	Max. acceleration = $49 \times \text{max. } x = 49 \times 0.4 = 19.6 \text{ m s}^{-2}$	B1 (1)
(c)	<p>String slack when $x = -0.2$: $v^2 = 49(0.4^2 - 0.2^2)$</p> <p>$\Rightarrow v \approx 2.42 \text{ m s}^{-1} = \frac{7\sqrt{3}}{5}$</p>	M1 A1 A1 (3)
(d)	<p>Uses $x = a \cos \omega t$ or use $x = a \sin \omega t$ but not with $x = 0$ or $\pm a$</p> <p>Attempt complete method for finding time when string goes slack $-0.2 = 0.4 \cos 7t \Rightarrow \cos 7t = -\frac{1}{2}$</p> <p>$t = \frac{2\pi}{21} \approx 0.299 \text{ s}$</p> <p>Time when string is slack = $\frac{(2) \times 2.42}{g} = \frac{2\sqrt{3}}{7} \approx 0.495 \text{ s}$ (2 needed for A)</p> <p>Total time = $2 \times 0.299 + 0.495 \approx 1.09 \text{ s}$</p>	M1 dM1 A1 A1 M1 A1ft A1 (7) 16
(a)	<p>DM1 requires the minus sign. Special case $2\ddot{x} = 2g - 98x$ is M1A1A0M0A0 $2\ddot{x} = -98x$ is M0A0A0M0A0</p>	
(b)	No use of \ddot{x} , just a is M1 A0,A0 then M1 A0 if otherwise correct. Quoted results are not acceptable.	
(c)	Answer must be positive and evaluated for B1	
(d)	<p>M1 – Use correct formula with their ω, a and x but not $x = 0$. A1 Correct values but allow $x = +0.2$ Alternative It is possible to use energy instead to do this part</p> <p>$\frac{1}{2}mv^2 + mg \times 0.6 = \frac{\lambda \times 0.6^2}{2l}$ M1 A1</p> <p>If they use $x = a \sin \omega t$ with $x = \pm 0.2$ and add $\frac{\pi}{7}$ or $\frac{\pi}{14}$ this is dM1, A1 if done correctly If they use $x = a \cos \omega t$ with $x = -0.2$ this is dM1, then A1 (as in scheme) If they use $x = a \cos \omega t$ with $x = +0.2$ this needs <i>their</i> $\frac{\pi}{7}$ minus answer to reach dM1, then A1</p>	