

Mark Scheme (Results)

January 2007

GCE

GCE Mathematics

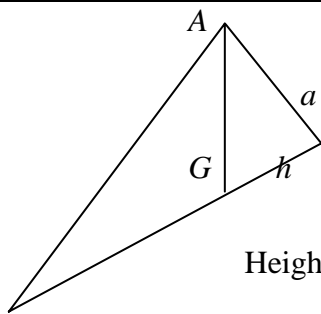
Mechanics M3 (6679)



January 2007
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) Maximum speed when accel. = 0 (o.e.)</p> <p>(b) $\frac{1}{12}(30 - x) = v \frac{dv}{dx}$ (acceln = ... + attempt to integrate)</p> <p>Use of $v \frac{dv}{dx}$: $\frac{v^2}{2} = \frac{1}{12} \left(30x - \frac{x^2}{2} \right) (+ c)$</p> <p>Substituting $x = 30$, $v = 10$ and finding $c (= 12.5)$, or limits</p> <p style="text-align: center;"><u>$v^2 = 25 + 5x - \frac{1}{12}x^2$</u> (o.e.)</p> <p>(a) Allow “acceln > 0 for $x < 30$, acceln < 0 for $x > 30$” Also “accelerating for $x < 30$, decelerating for $x > 30$” But “acceln < 0 for $x > 30$” only is B0</p> <p>(b) 1st M1 will be generous for wrong form of acceln (e.g. dv/dx)! 3rd M1 If use limits, they must use them in correct way with correct values Final A1. Have to accept any expression, but it must be for v^2 explicitly (not $1/2v^2$), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above, or e.g. $\frac{1}{12}(300 + 60x - x^2)$; also $100 - \frac{1}{12}(30 - x)^2$</p>	<p>B1 (1)</p> <p>M1 ↓ M1 A1 ↓ M1 A1 (5)</p>

2.



$$\text{Height of cone} = \frac{a}{\tan \alpha} = 3a$$

$$\text{Hence } h = \frac{3}{4}a$$

$$\tan \theta = \frac{a}{\frac{3}{4}a} = \frac{4}{3} \Rightarrow \theta = 53.1^\circ$$

1st M1 (generous) allow any trig ratio to get height of cone (e.g. using sin)

3rd M1 For correct trig ratio on a suitable triangle to get θ or complement (even if they call the angle by another name – hence if they are aware or not that they are getting the required angle)

M1 A1

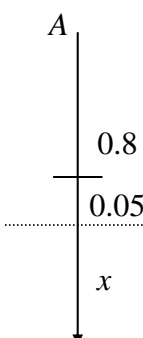
↓
M1↓
M1 A1
(5)

3.	<p>(a) $\text{E.P.E.} = \frac{1}{2} \frac{3.6mg}{a} x^2 = \frac{1}{2} \frac{3.6mg}{a} \left(\frac{a}{3}\right)^2$ $= \underline{0.2 mga}$</p> <p>(b) Friction = $\mu mg \Rightarrow$ work done by friction = $\mu mg \left(\frac{4a}{3}\right)$</p> <p>Work-energy: $\frac{1}{2} m \cdot 2ga = \mu mgd + 0.2 mga$ (3 relevant terms)</p> <p>Solving to find μ: <u>$\mu = 0.6$</u></p> <p>(b) 1st M1: allow for attempt to find work done by frictional force (i.e. not just finding friction). 2nd M1: “relevant” terms, i.e. energy or work terms! A1 f.t. on their work done by friction</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1 A1√ ↓</p> <p>M1 A1 (6)</p>
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4.	<p>(a) Energy: $\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1 + \cos \theta)$</p> $\underline{v^2 = ag(1 - 2\cos \theta)} \quad (\text{o.e.})$ <p>(b) $T + mg \cos \theta = m \frac{v^2}{a}$</p> <p>Hence $\underline{T = (1 - 3\cos \theta)mg}$ (*)</p> <p>(c) Using $T = 0$ to find $\cos \theta$</p> <p>Hence height above A = $\underline{\frac{4}{3}a}$ Accept $1.33a$ (but must have 3+ s.f.)</p> <p>(d) $v^2 = \frac{1}{3}ag$ (o.e.) f.t. using $\cos \theta = \frac{1}{3}$ in v^2</p> <p>consider vert motion: $(v \sin \theta)^2 = 2gh$ (with v resolved)</p> <p>$\sin^2 \theta = \frac{8}{9}$ (or $\theta = 70.53$, $\sin \theta = 0.943$) and solve for h (as ka)</p> $h = \underline{\frac{4}{27}a}$ or $0.148a$ (awrt) <p>OR consider energy: $\frac{1}{2}m(v \cos \theta)^2 + mgh = \frac{1}{2}mv^2$ (3 non-zero terms)</p> <p>Sub for v, θ and solve for h</p> $h = \underline{\frac{4}{27}a}$ or $0.148a$ (awrt)	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>A1 cso (3)</p> <p>M1</p> <p>A1 (2)</p> <p>B1√</p> <p>M1 A1 ↓ M1</p> <p>A1</p> <p>M1 A1 ↓ M1</p> <p>A1</p>
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Question Number	Scheme	Marks
5.	<p>(a) $\updownarrow T \cos \theta = mg$</p> <p>$\leftrightarrow T + T \sin \theta = m r \omega^2$ (3 terms)</p> <p>$r = h \tan \theta$</p> <p>$\frac{mg}{\cos \theta} (1 + \sin \theta) = \frac{m \omega^2 h \sin \theta}{\cos \theta}$ (eliminate r)</p> <p>$\omega^2 = \frac{g}{h} \left(\frac{1 + \sin \theta}{\sin \theta} \right)$ (*) (solve for ω^2)</p> <p>(b) $\omega^2 = \frac{g}{h} \left(\frac{1}{\sin \theta} + 1 \right) > \frac{2g}{h} (\sin \theta < 1) \Rightarrow \omega > \sqrt{\frac{2g}{h}}$ (*)</p> <p>(c) $\frac{3g}{h} = \frac{g}{h} \left(\frac{1 + \sin \theta}{\sin \theta} \right) \Rightarrow \sin \theta = \frac{1}{2}$</p> <p>$T \cos \theta = mg \Rightarrow T = \frac{2\sqrt{3}}{3} mg$ or <u>1.15mg</u> (awrt)</p> <p>(a) Allow first B1 M1 A1 if assume different tensions (so next M1 is effectively for eliminating r and T.)</p> <p>(b) M1 requires a <i>valid</i> attempt to derive an <i>inequality</i> for ω. (Hence putting $\sin \theta = 1$ immediately into expression of ω^2 [assuming this is the critical value] is M0.)</p>	<p>B1</p> <p>M1 A1</p> <p>B1</p> <p>↓ M1</p> <p>↓ M1 A1 (7)</p> <p>M1 A1 (2)</p> <p>M1 A1</p> <p>↓ M1 A1 (4)</p>

<p>6.</p>	<p>(a) Moments: $\pi \int_1^2 xy^2 dx = V \bar{x}$ or $\int_1^2 xy^2 dx = \bar{x} \int_1^2 y^2 dx$</p> $\int_1^2 y^2 dx = \int_1^2 \frac{1}{4x^4} dx = \left[-\frac{1}{12x^3} \right]_1^2 \quad (= \frac{7}{96}) \quad \text{(either)}$ $\int_1^2 xy^2 dx = \int_1^2 \frac{1}{4x^3} dx = \left[-\frac{1}{8x^2} \right]_1^2 \quad (= \frac{3}{32}) \quad \text{(both)}$ <p>Solving to find \bar{x} ($= \frac{9}{7}$) \Rightarrow required dist = $\frac{9}{7} - 1 = \frac{2}{7}$ m (*)</p> <p>(b)</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%;"></td> <td style="text-align: center;"><i>H</i></td> <td style="text-align: center;"><i>S</i></td> <td style="text-align: center;"><i>T</i></td> </tr> <tr> <td>Mass</td> <td style="text-align: center;">$(\rho) \frac{2}{3} \pi \left(\frac{1}{2}\right)^3$</td> <td style="text-align: center;">$(\rho) \frac{7\pi}{96}$</td> <td style="text-align: center;">$H + S$</td> </tr> <tr> <td></td> <td style="text-align: center;">$\left[= \frac{1}{12}(\rho)\pi \right]$</td> <td></td> <td style="text-align: center;">$\left[= \frac{5}{32}(\rho)\pi \right]$</td> </tr> </table> <p>Dist of CM from base $\frac{19}{16}$ m $\frac{5}{7}$ m \bar{x}</p> <p>Moments: $\left[= \frac{1}{12}(\rho)\pi \right] \left(\frac{19}{16}\right) + (\rho) \frac{7\pi}{96} \left(\frac{5}{7}\right) = \left[\frac{5}{32}(\rho)\pi \right] \bar{x}$</p> <p style="text-align: center;">$\bar{x} = \frac{29}{30}$ m or 0.967 m (awrt)</p> <p>Allow distances to be found from different base line if necessary</p>		<i>H</i>	<i>S</i>	<i>T</i>	Mass	$(\rho) \frac{2}{3} \pi \left(\frac{1}{2}\right)^3$	$(\rho) \frac{7\pi}{96}$	$H + S$		$\left[= \frac{1}{12}(\rho)\pi \right]$		$\left[= \frac{5}{32}(\rho)\pi \right]$	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p>↓</p> <p>M1 A1 cso (6)</p> <p>B1, M1</p> <p>B1 B1</p> <p>M1 A1</p> <p>A1 (7)</p>
	<i>H</i>	<i>S</i>	<i>T</i>											
Mass	$(\rho) \frac{2}{3} \pi \left(\frac{1}{2}\right)^3$	$(\rho) \frac{7\pi}{96}$	$H + S$											
	$\left[= \frac{1}{12}(\rho)\pi \right]$		$\left[= \frac{5}{32}(\rho)\pi \right]$											

<p>7.</p>	<p>(a) </p> $T = \frac{\lambda}{0.8}(0.05) = 0.25g$ $\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2 \text{ (*)}$ <p>(b)</p> $T = \frac{39.2}{0.8}(x + 0.05)$ $mg - T = ma \quad \text{(3 term equn)}$ $0.25g - \frac{39.2}{0.8}(x + 0.05) = 0.25 \ddot{x} \text{ (or equivalent)}$ $\ddot{x} = -196x$ <p>SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7} \text{ s (*)}$</p> <p>(c)</p> $v = 14 \sqrt{\{(0.1)^2 - (0.05)^2\}}$ $= 1.21(24\dots) \approx \underline{1.21 \text{ m s}^{-1}} \text{ (3 s.f.) Accept } 7\sqrt{3}/10$ <p>(d) Time T under gravity = $\frac{1.21..}{g} (= 0.1237 \text{ s})$ Complete method for time T' from B to slack. [↑ e.g. $\frac{\pi}{28} + t$, where $0.05 = 0.1 \sin 14t$ OR T', where $-0.05 = 0.1 \cos 14T'$] $T'' = 0.1496 \text{ s}$ Total time = $T + T' = \underline{0.273 \text{ s}}$</p> <p>(b) 1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if e.g. $x + 0.15$), or must justify later For last four marks, <i>must</i> be using \ddot{x} (not a)</p> <p>(c) Using $x = 0$ is M0 (d) M1 – must be using distance for when string goes slack. Using $x = -0.1$ (i.e. assumed end of the oscillation) is M0</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1 (3 term equn)</p> <p>A1</p> <p>A1 ↓ M1 A1 cso (6)</p> <p>M1 A1√</p> <p>A1 (3)</p> <p>B1√</p> <p>M1 A1</p> <p>A1</p> <p>A1 (5)</p>
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