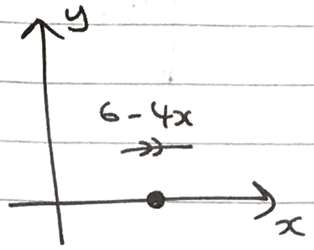
M3 Mock (MA)

Q1)



$a = 6 - 4x$

$v\frac{dv}{dx} = 6 - 4x$

$\int (v)\, dv = \int (6-4x)\, dx$

$\frac{v^2}{2} = 6x - 2x^2 + c$

$v^2 = 12x - 4x^2 + d.$

$x = 0, v = 4$: $16 = d$ //

$\therefore v^2 = 12x - 4x^2 + 16$

at $v = 0$: $-4x^2 + 12x + 16 = 0$

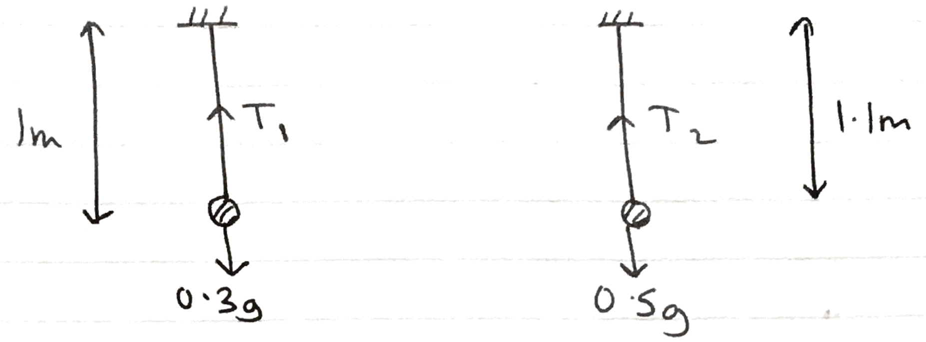
$4x^2 - 12x - 16 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$ as $(x > 0)$

Q2a)



$T_1 = 0 \cdot 3g$

$T_1 = \dfrac{\lambda(1-l)}{l} = 0 \cdot 3g \longrightarrow \lambda = \dfrac{0 \cdot 3gl}{1-l}$ ~ ①

$T_2 = 0 \cdot 5g$

$T_2 = \dfrac{\lambda(1 \cdot 1-l)}{l} = 0 \cdot 5g \longrightarrow \lambda = \dfrac{0 \cdot 5gl}{1 \cdot 1-l}$ ~ ②

① = ② : $\dfrac{0 \cdot 3\cancel{g}\cancel{l}}{1-l} = \dfrac{0 \cdot 5\cancel{g}\cancel{l}}{1 \cdot 1-l}$

$0 \cdot 3(1 \cdot 1 - l) = 0 \cdot 5(1-l)$

$0 \cdot 33 - 0 \cdot 3l = 0 \cdot 5 - 0 \cdot 5l$

$0 \cdot 2l = 0 \cdot 17$

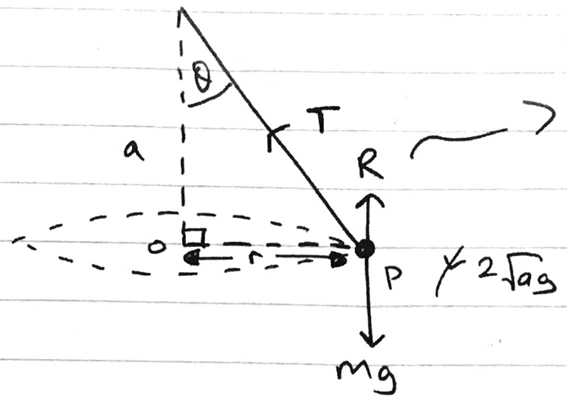
$\therefore l = \dfrac{0 \cdot 17}{0 \cdot 20} = \boxed{\dfrac{17}{20}}\ m. \quad = 0 \cdot 85m = 85cm$

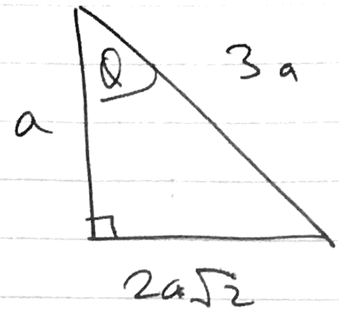
b) from ②, $\dfrac{\lambda(1 \cdot 1 - l)}{l} = 0 \cdot 5g$

$\longrightarrow \lambda = \dfrac{0 \cdot 5g\left(\frac{17}{20}\right)}{1 \cdot 1 - 17/20} = \boxed{16 \cdot 66N}$

$$\sqrt{(3a)^2-(a)^2} = 2a\sqrt{2} = r$$

(Q3a)





$\overleftarrow{+}$ N2L(P) : $T\sin\theta = \frac{mv^2}{r}$

$$T\left(\frac{2\sqrt{2}}{3}\right) = \frac{m}{2\cancel{a}\sqrt{2}}(4\cancel{a}g)$$

$$T\left(\frac{2\sqrt{2}}{3}\right) = \frac{4mg}{2\sqrt{2}}$$

$$\frac{8T}{3} = 4mg$$

$$\therefore T = 4mg \times \frac{3}{8} = \boxed{\frac{3mg}{2}}$$

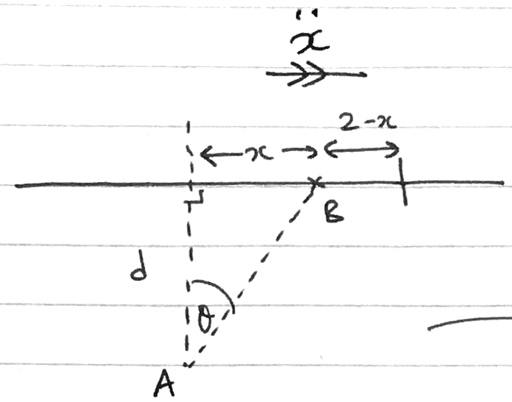
b) $R(\updownarrow)$: $T\cos\theta + R = mg$

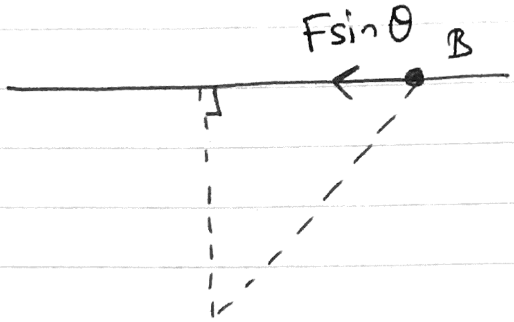
$$\frac{3mg}{2}\left(\frac{1}{3}\right) + R = mg$$

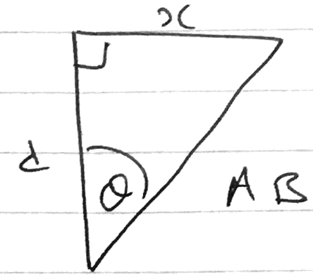
$$R = mg - \frac{mg}{2} = \boxed{\frac{mg}{2}}$$

B will move along the wire subject to a force of magnitude $5AB\sin\theta$ directed towards O.

(Q4a)







$\overrightarrow{N2L}^{+}$ (B) : $-F\sin\theta = 0.2\ddot{x}$

$F = 5AB$

$\therefore -5AB \times \dfrac{x}{AB} = 0.2\ddot{x}$ $\qquad \sin\theta = \dfrac{x}{AB}$

$\Rightarrow \quad -5x = 0.2\ddot{x}$

$\Rightarrow \quad -25x = \ddot{x}$

$\omega^2 = 25$

$\therefore \omega = 5$

$T = \dfrac{2\pi}{\omega} = \boxed{\dfrac{2\pi}{5}}$

b) $a = OB = 2m$

$$V_{max} = a\omega = 2 \times 5 = \boxed{10ms^{-1}}$$

c) when B has moved a distance of 3m, $x = -1$.

$$x = a\cos\omega t$$
$$x = 2\cos 5t$$

$$-1 = 2\cos 5t$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 5t$$

$$\therefore t = \frac{1}{5}\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{15}}\ s.$$

Q5a) Initially : $KE = \frac{1}{2}(2)(10)^2 = 100J.$

$GPE = 0$
$EPE = \frac{120}{6}(0) = 0$

At rest : $EPE = \frac{120}{6}(d)^2$
$KE = 0$
$GPE = 2gd$

where D is vertical distance moved by the ball.

C.O.E : $100 = 20d^2 + 19.6d$

$$20d^2 + 19.6d - 100 = 0$$

By Quadratic formula, $\boxed{d = 1.8\text{m}}$ (as $d > 0$.)

b) assume ball comes to rest after travelling 1.8m.

Initially : $KE = v^2$
$GPE = 0$
$EPE = \frac{120}{6}(0.5)^2$

At rest : $KE = 0$
(highest point) $GPE = 2g(1.8)$
$EPE = \frac{120}{6}(d+0.5)^2 \rightarrow (d = 1.8)$

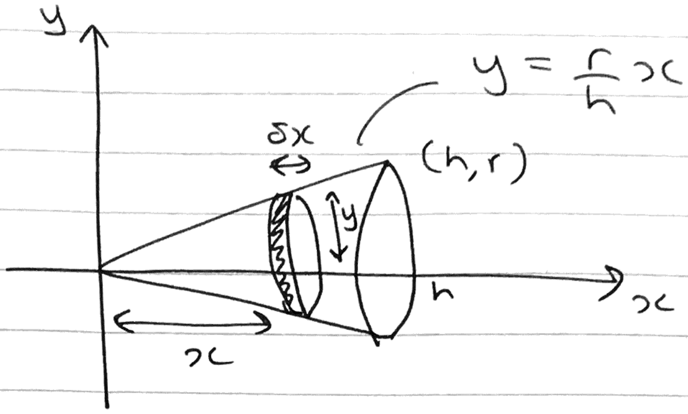
C.O.E : $V^2 + 20\left(\frac{1}{4}\right) = 2g(1.8) + 20\left(d+\frac{1}{2}\right)^2$

$V^2 + 5 = 35.28 + 20(1.8+0.5)^2$

$V^2 = 35.28 - 5 + 20(2.3)^2$

$V = \sqrt{136.08} \approx \boxed{11.7\text{ms}^{-1}}$

Q6a)



splitting up the cone into an infinite amount of 'thin' discs each of thickness δx.

mass of entire cone = $m = \rho \times$ volume

$m = \rho \times \frac{1}{3}\pi r^2 h$

mass of one disc = $\delta m = \rho \times [\pi y^2 \delta x]$

distance of c.o.m of one disc from O $= x$.

recall from M2 that $\bar{x} \sum m_i = \sum m_i x_i$

$$\Rightarrow \bar{x}\left(\frac{\rho\pi r^2 h}{3}\right) = \sum_{x=0}^{h}[\rho\pi y^2 x\delta x]$$

$$\lim_{\delta x \to 0} \sum_{x=0}^{h} \rho\pi y^2 x\,\delta x = \rho\pi \int_0^h (y^2 x)\,dx$$

$$\rho\pi \int_0^h (y^2 x)\,dx = \rho\pi \int_0^h \left(\frac{r^2 x^3}{h^2}\right) dx$$

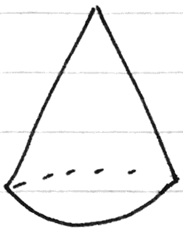
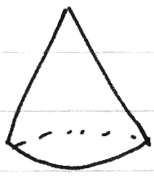
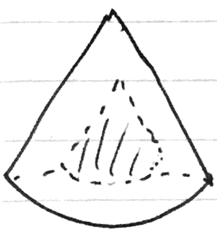
$$= \frac{\rho\pi r^2}{h^2}\left[\frac{x^4}{4}\right]_0^h = \frac{\rho\pi r^2 h^2}{4}$$ //

so $\frac{p\pi r^2 h^2}{4} = \left(\frac{ph\pi r^2}{3}\right)\bar{x}$

$\xrightarrow{\div p\pi r^2 h}$ $\frac{h}{4} = \frac{\bar{x}}{3}$

$\xrightarrow{\times 3}$ $\bar{x} = \frac{3h}{4}$

b)

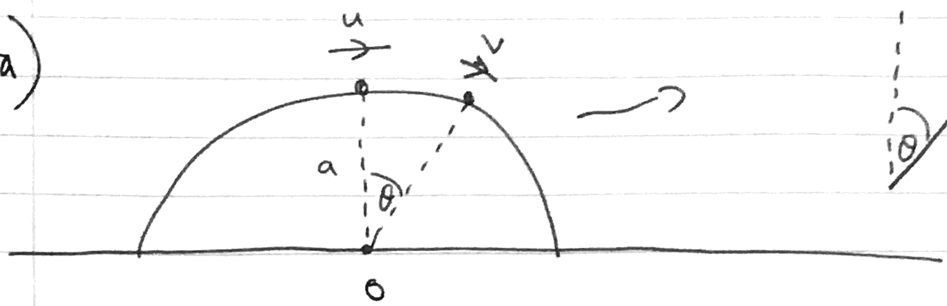
	Shape	Mass (vol.)	Distance of c.o.m from A
+		$\frac{1}{3}\pi(a)^2(h) = \boxed{\frac{\pi a^2 h}{3}}$	$\boxed{\frac{3h}{4}}$
−		$\frac{1}{3}\pi\left(\frac{2a}{3}\right)^2\left(\frac{h}{2}\right) = \boxed{\frac{2}{27}\pi a^2 h}$	$\frac{3}{4}\left(\frac{h}{2}\right) + \frac{h}{2} = \boxed{\frac{7h}{8}}$
=		$\boxed{\frac{7}{27}\pi a^2 h}$	$\boxed{\bar{y}}$

taking moments about A . . .

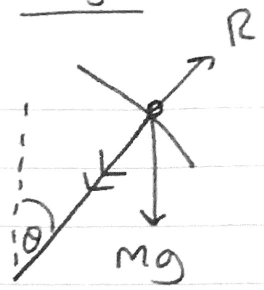
$$\frac{\cancel{\pi a^2 h}}{3}\left(\frac{3h}{4}\right) - \frac{2}{27}\cancel{\pi a^2 h}\left(\frac{7h}{8}\right) = \frac{7}{27}\cancel{\pi a^2 h}\,(\bar{y})$$

$$\frac{\frac{h}{4} - \frac{7}{8}\left(\frac{2h}{27}\right)}{\frac{7}{27}} = \bar{y} = \frac{\frac{5h}{27}}{\frac{7}{27}} = \boxed{\frac{5h}{7}}$$

Q7a)



at angle θ :



initially at top: KE = $\frac{1}{2}mu^2$

GPE = mga

at angle θ to vertical : KE = $\frac{1}{2}mv^2$

GPE = $mga\cos\theta$.

C.O.E : $\frac{mu^2}{2} + amg = \frac{mv^2}{2} + amg\cos\theta$

$\Rightarrow \frac{u^2}{2} + ag(1-\cos\theta) = \frac{v^2}{2}$

$\Rightarrow v^2 = u^2 + 2ag(1-\cos\theta)$

$\Rightarrow v^2 = \frac{ag}{2} + 2ag(1-\cos\theta)$

$\Rightarrow v^2 = ag\left(\frac{5}{2} - 2\cos\theta\right) = \frac{ag}{2}(5-4\cos\theta)$

b) N2L(P) : $mg\cos\theta - R = \frac{m}{a}v^2$

$$R = mg\cos\theta - \frac{m}{a}v^2$$

$$R = mg\cos\theta - \frac{m}{a}(ag)\left(\frac{5}{2} - 2\cos\theta\right)$$

$$R = mg\cos\theta - \frac{5}{2}mg + 2mg\cos\theta$$

$$R = 3mg\cos\theta - \frac{5}{2}mg$$

at $\theta = \arccos(0.9)$, $\cos\theta = 0.9$

$$\Rightarrow R = 3mg \times 0.9 - 2.5mg = 0.2mg > 0$$

$R > 0$ $\therefore$ P is still on the hemisphere.

c) $R = 0$: $mg\cos\theta - \frac{m}{a}v^2 = 0$

$$v^2 = ag\cos\theta$$

and $v^2 = \frac{ag}{2}(5 - 4\cos\theta)$

$$\Rightarrow \frac{\cancel{ag}}{2}(5 - 4\cos\theta) = \cancel{ag}\cos\theta$$

$$\frac{5}{2} - 2\cos\theta = \cos\theta$$

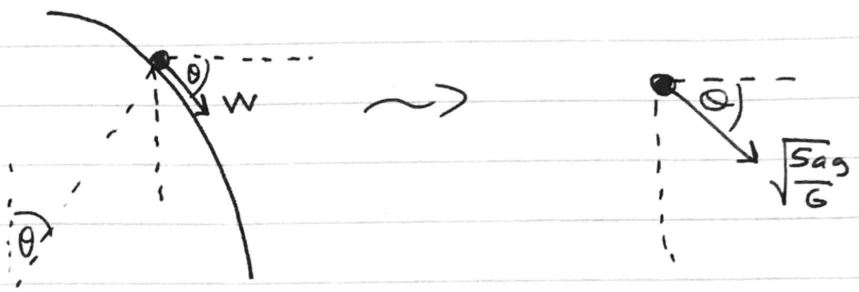
$$3\cos\theta = \frac{5}{2} \quad \therefore \cos\theta = \frac{5}{6}$$

d) $v^2 = ag\left(\frac{5}{2} - 2\cos\theta\right)$

$$\cos\theta = \frac{5}{6} \therefore \quad v^2 = ag\left(2.5 - \frac{10}{6}\right) = \frac{5ag}{6}$$

$$\therefore v = \sqrt{\frac{5ag}{6}}$$

e)



so $\vec{w} = \cos\theta\sqrt{\frac{5ag}{6}} = \frac{5}{6}\sqrt{\frac{5ag}{6}}$

and $\downarrow w = \sin\theta\sqrt{\frac{5ag}{6}} = \frac{\sqrt{11}}{6}\cdot\sqrt{\frac{5ag}{6}} = \frac{1}{6}\sqrt{\frac{55ag}{6}}$

$\sin\theta = \sqrt{1-\left(\frac{5}{6}\right)^2} = \frac{\sqrt{11}}{6}$

$(\sin\theta = \sqrt{1-\cos^2\theta})$

$\boxed{w = \text{initial speed when leaving the hemisphere}}$

<u>till P hits the ground</u> :

$$\left.\begin{aligned} s &= \frac{5a}{6} \\ u &= \frac{1}{6}\sqrt{\frac{55ag}{6}} \\ v &= v \\ a &= g \\ t &= \end{aligned}\right\} \quad \downarrow +$$

$v^2 = u^2 + 2as$

$$v^2 = \frac{55ag}{216} + \frac{5ag}{3} = \frac{415ag}{216}$$

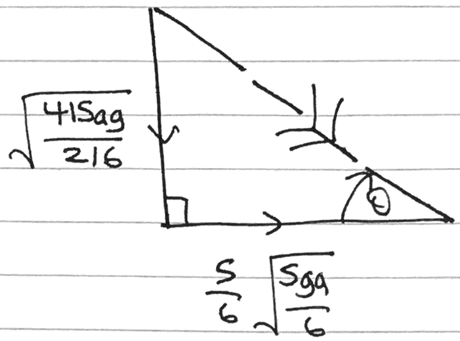
and $\vec{v} = \vec{w} = \frac{5}{6}\sqrt{\frac{5ga}{6}}$

(no $\overrightarrow{\text{acceleration}}$)

$\therefore$ speed at B $= \sqrt{\frac{415ag}{216} + \left(\frac{5}{6}\sqrt{\frac{5ga}{6}}\right)^2} = \boxed{\sqrt{\frac{5ga}{2}}}$

$= (4.95\sqrt{a})$ m/s

f)



$\tan\theta = \dfrac{\sqrt{\frac{415}{216}}}{\frac{5}{6}\sqrt{\frac{5}{6}}} = 1.822...$

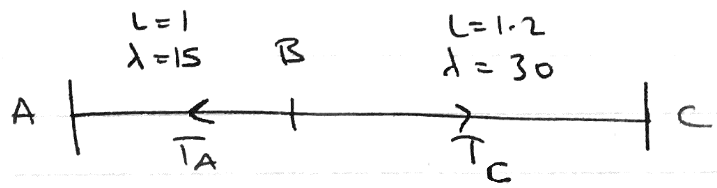
$\theta = \tan^{-1}(1.822) = \boxed{61.2^\circ}$

to the horizontal

d) alt : $\vec{u}$ = horizontal component initially $= \sqrt{\frac{ag}{2}}$ from (c)
then suvat to find $\uparrow w$ and use pythagoras...
(ie $\sqrt{\uparrow w^2 + \vec{u}^2} = v$)

e) alt : $\cos\theta = \dfrac{\text{horizontal speed}}{\text{speed from (d)}}$

Alternative Question 2



the strings are in equilibrium
so $T_A = T_C$

$$\Rightarrow \quad \frac{15}{1}(AB - 1) = \frac{30}{1.2}(3 - AB - 1.2)$$

$$\Rightarrow \quad 15AB - 15 = 25(1.8 - AB)$$

$$\Rightarrow \quad 15AB - 15 = 45 - 25AB$$

$$\Rightarrow \quad 40AB = 60$$

$$\Rightarrow \quad AB = 1.5\text{m} \quad (= BC)$$

so $T_A = \frac{15}{1}(1.5 - 1) = \boxed{7.5N}$ = combined tension.