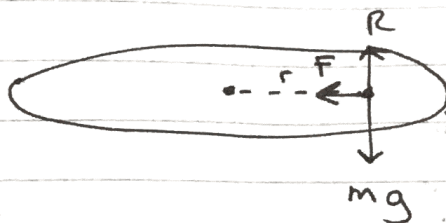


M3 June 2018 (MA)

Q1)



$$\underline{N2L(P)} : F = ma$$

$$F = mr\omega^2$$

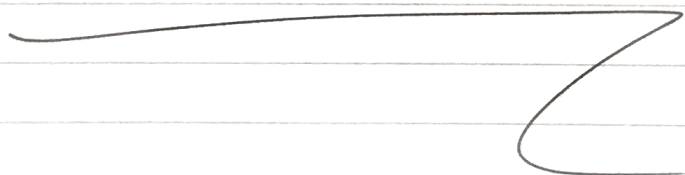
but  $F \leq \mu R$  since P doesn't slip...

$$\therefore F \leq \mu mg \quad (\text{since } R = mg)$$

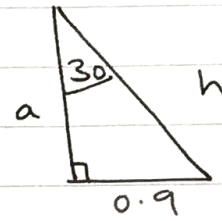
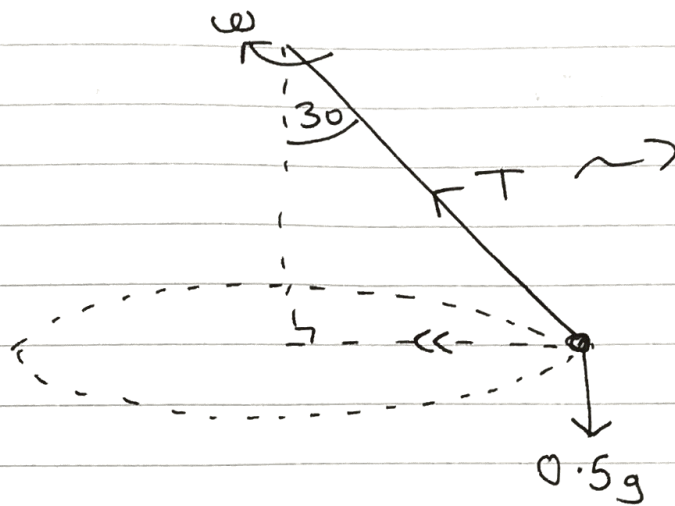
$$\Rightarrow mr\omega^2 \leq \mu mg$$

$$\omega^2 \leq \frac{\mu g}{r}$$

$$\text{hence } \omega \leq \sqrt{\frac{\mu g}{r}}$$



Q2)  
a)



$$\tan 30 = \frac{0.9}{a}$$

$$a = \frac{0.9}{\tan 30} = \frac{9\sqrt{3}}{10}$$

$$\therefore h = \sqrt{\left(\frac{9\sqrt{3}}{10}\right)^2 + (0.9)^2}$$

$$h = 1.8$$

≡

$$R(\uparrow\downarrow): T \cos 30 = 0.5g$$

$$T = \frac{\lambda x}{l} = \lambda \left( \frac{1.8 - 1.2}{1.2} \right)$$

$$\therefore \lambda \left( \frac{1}{2} \right) \cos 30 = 0.5(9.8)$$

$$\lambda = \frac{0.5 \times 9.8}{\frac{1}{2} \times \cos 30} = \frac{98\sqrt{3}}{15} = \boxed{11.3}$$

b) N2L (particle):  $T \sin 30 = m r \omega^2$

$$\frac{T}{2} = 0.5(0.9)\omega^2$$

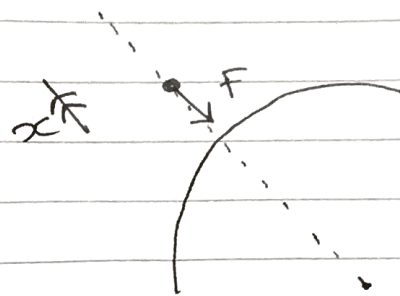
$$\frac{T}{0.9} = \omega^2 = \frac{\lambda x}{0.9} = \frac{(11.3)(0.6)}{0.9(1.2)}$$

$$= 6.287 \dots$$

$$\text{so } \omega = \sqrt{6 \cdot 287 \dots} = \boxed{2.51}$$

(Q3.)  $-F = ma$

$$-\frac{mgR^2}{x^2} = m v \frac{dv}{dx}$$



$$\int (v) dv = -gR^2 \int (x^{-2}) dx$$

$$\frac{v^2}{2} = -gR^2 \left[ -\frac{1}{x} \right] + c$$

$$\frac{v^2}{2} = \frac{gR^2}{x} + c$$

$$v^2 = \frac{2gR^2}{x} + c'$$

$$x = 3R, v = \sqrt{\frac{gR}{3}} : \frac{gR}{3} = \frac{2gR^2}{3R} + c'$$

$$c' = \frac{gR}{3} - \frac{2gR}{3} = -\frac{gR}{3}$$

$$\therefore v^2 = \frac{2gR^2}{x} - \frac{gR}{3}$$

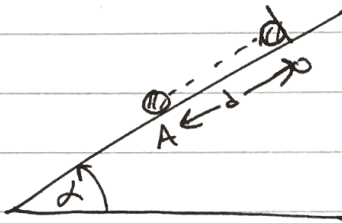
$$v = 2\sqrt{\frac{gR}{3}} : \frac{4gR}{3} = -\frac{gR}{3} + \frac{2gR^2}{x}$$

$$\frac{5}{3} = \frac{2R}{x}$$

$$\frac{5}{3}x = 2R \rightarrow x = \frac{6R}{5}$$

$$\therefore \text{distance from surface of earth} \\ = \frac{6R}{5} - R = \boxed{\frac{R}{5}}$$

Q4)



$$\sin \alpha = \frac{3}{5} \rightarrow \cos \alpha = \frac{4}{5}$$

$$\mu = \frac{1}{4}$$

InitiallyFinally

$$KE = 0$$

$$KE = 0$$

$$GPE = mgd \sin \alpha$$

$$GPE = 0$$

$$EPE = 0$$

$$EPE = \frac{2mg}{2L} (d-L)^2$$

$$\begin{aligned} \text{W.D due to friction} &= \mu R d = \frac{1}{4} (mg \cos \alpha) (d) \\ &= \frac{mgd}{5} \end{aligned}$$

$$\overset{\text{C.O.E}}{\Rightarrow} mgd \left( \frac{3}{5} \right) = \frac{mg}{L} (d-L)^2 + \frac{mgd}{5}$$

$$\Rightarrow \frac{2mgd}{5} = \frac{mg}{L} (d^2 - 2dL + L^2)$$

$$\frac{2dl}{5} = d^2 - 2dl + l^2$$

$$d^2 - \frac{12dl}{5} + l^2 = 0$$

By Quadratic Formula...

$$\left. \begin{array}{l} a = 1 \\ b = -\frac{12l}{5} \\ c = l^2 \end{array} \right\} d = \frac{\frac{12l}{5} \pm \sqrt{\left(\frac{12l}{5}\right)^2 - 4l^2}}{2}$$

$$d = \frac{\frac{12l}{5} \pm \sqrt{\frac{44}{25}l^2}}{2}$$

$$d = \frac{\frac{12}{5}l \pm 1.327l}{2}$$

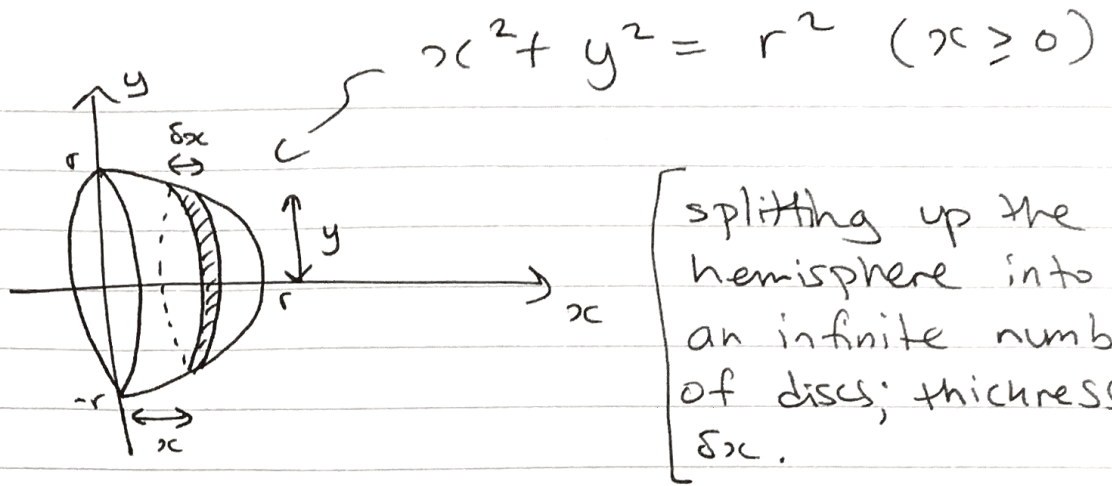
$$d > l \quad \text{so} \quad d = l \left( \frac{12}{5} + 1.327 \right)$$

$$\therefore k = \frac{12}{5} + 1.327$$

$$\approx \boxed{1.86}$$

11

Q5a)



$$\frac{\text{Mass}}{\text{Volume}} = \frac{m}{\frac{2}{3}\pi r^3} = \frac{3m}{2\pi r^3} \quad \text{for entire hemisphere.}$$

So for one disc,  $\frac{\text{mass}}{\frac{2}{3}\pi r^3} = \frac{3m}{2\pi r^3} \times \pi y^2 \delta x$

$$m = \frac{3my^2 \delta x}{2r^3} //$$

recall from M2 that  $\bar{x} \sum m_i = \sum m_i x_i$

$$\Rightarrow \bar{x}(m) = \sum_{x=0}^r \frac{3my^2 \delta x}{2r^3} \times x$$

$$\lim_{\delta x \rightarrow 0} \sum_0^r \frac{3mx y^2 \delta x}{2r^3} = \frac{3m}{2r^3} \int_0^r (x y^2) dx$$

$$\Rightarrow \frac{3m}{2r^3} \int_0^r x (r^2 - x^2) dx = \frac{3m}{2r^3} \int_0^r (r^2 x - x^3) dx$$


$$= \frac{3m}{2r^3} \left[ \frac{r^2 x^2}{2} - \frac{x^4}{4} \right]_0^r = \frac{3m}{2r^3} \left[ \frac{r^4}{2} - \frac{r^4}{4} \right] = \frac{3mr^4}{2(4r^3)}$$

$$= \frac{3mr}{2 \times 4} //$$

$$\text{so } \bar{r} (m) = \frac{3mr}{8}$$

$$\text{hence } \bar{r} = \frac{3r}{8}$$

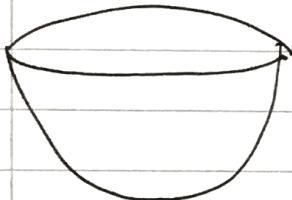
b) Shape      Mass (Vol.)      Displacement of c.o.m from O



$$\frac{2}{3}\pi\left(\frac{a}{2}\right)^3 \times 4$$

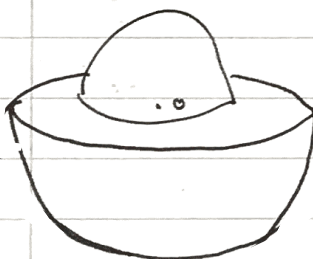
$$= \boxed{\frac{2\pi u a^3}{24}}$$

$$-\frac{3}{8}\left(\frac{a}{2}\right) = \boxed{-\frac{3a}{16}}$$



$$\boxed{\frac{2}{3}\pi a^3}$$

$$\boxed{\frac{3a}{8}}$$



$$\boxed{\pi a^3 \left[ \frac{2}{3} + \frac{2u}{24} \right]}$$

$$\boxed{\bar{y}}$$

Moments about a diameter through O

$$\frac{u}{12} \left( -\frac{3a}{16} \right) + \frac{2}{3} \left( \frac{3a}{8} \right) = \left( \frac{2}{3} + \frac{u}{12} \right) \bar{y}$$

$$-\frac{3ua}{192} + \frac{a}{4} = \left( \frac{2}{3} + \frac{u}{12} \right) \bar{y}$$

$$\bar{y} = \frac{a \left( \frac{1}{4} - \frac{3u}{192} \right) \times 192}{\left( \frac{2}{3} + \frac{u}{12} \right) \times 192}$$

$$= \frac{a(48 - 3u)}{128 + 16u} //$$

$$\therefore \underline{\text{distance}} = \frac{a(48 - 3u)}{16(u + 8)}$$

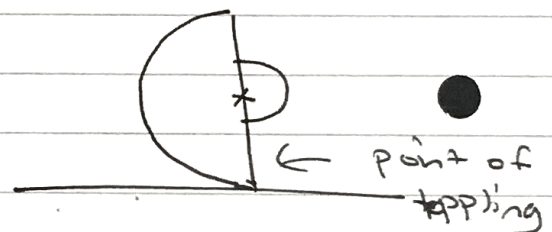
c) the information given means that the centre of mass is at 0.

$$\text{so } \bar{y} = 0$$

$$a(48 - 3u) = 0$$

$$48 - 3u = 0$$

$$u = \frac{48}{3} = \boxed{16}$$





● (Q6a)

Initially

$$KE = \frac{1}{2} mu^2$$

$$GPE = 0$$

At angle  $\theta$  to horizontal

$$KE = \frac{1}{2} mv^2$$

$$GPE = mgsin\theta$$

C.O.E :  $\frac{mu^2}{2} = \frac{mv^2}{2} + mgsin\theta$

x2 :  $u^2 = v^2 + 2gsin\theta$

$$v^2 = u^2 - 2gsin\theta //$$



N2L (?) :  $T + mgsin\theta = \frac{mv^2}{r}$

$$T = \frac{m}{a} (u^2 - 2gsin\theta) - mgsin\theta$$

$$T = \frac{mu^2}{a} - 2mgsin\theta - mgsin\theta$$

$$T = \frac{mu^2}{a} - 3mgsin\theta$$

$$\therefore T = \frac{m}{a} (u^2 - 3gsin\theta)$$

b)  $T \geq 0$  at top.

min. value of  $u$  is when  $T = 0$  at top.

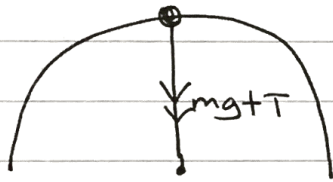
$$\Rightarrow \frac{m}{a} (u^2 - 3ag \sin \theta) = 0$$

$$\Rightarrow u^2 - 3ag \sin \theta = 0$$

at top,  $\theta = 90^\circ$  so  $u^2 = 3ag$  //

$$\text{hence } u_{\min} = \boxed{\sqrt{3ag}}$$

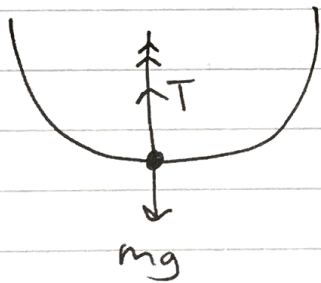
c)



Tension is minimum at top.

$$\theta = 90^\circ: T = S = \frac{m}{a} (u^2 - 3ag) //$$

~ ①



$\theta = 270^\circ$ : (At bottom tension is max)

$$T = 4S = \frac{m}{a} (u^2 + 3ag)$$

$$S = \frac{m}{4a} (u^2 + 3ag) //$$

~ ②

$$\text{①} = \text{②}: \frac{m}{a} (u^2 - 3ag) = \frac{m}{4a} (u^2 + 3ag)$$

$$u^2 - 3ag = \frac{u^2}{4} + \frac{3ag}{4}$$

$$\text{so } \frac{3u^2}{4} = ag \left( 3 + \frac{3}{4} \right)$$

$$u^2 = \frac{4ag}{3} \left( \frac{15}{4} \right)$$

$$u^2 = 5ag \quad \text{hence}$$

$$u = \sqrt{5ag}$$

(Q7a)



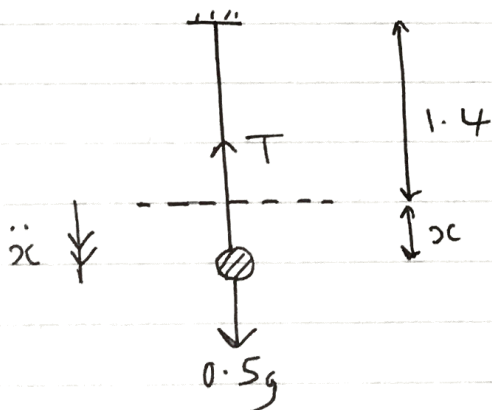
$$T = mg$$

$$\frac{29.4(1.4 - l)}{l} = 4.9$$

$$41.16 - 29.4l = 4.9l$$

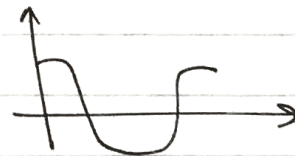
$$l = \frac{41.16}{29.4 + 4.9} = 1.2$$

b)



$x$  = distance from equilibrium (B) here.

P starts at an endpoint  
so  $x = a \cos \omega t$  applies



so  $x$  is max at  $t = 0$  which means  $x$  increases in the direction AB. so  $\ddot{x}$  is positive in this direction too.

$$\downarrow \text{N2L (P)}: 0.5g - T = 0.5\ddot{x}$$

$$g - 2T = \ddot{x}$$

$$g - 2\left(\frac{29.4}{1.2}\right)(x+0.2) = \ddot{x}$$

$$9.8 - 49(x) - 49(0.2) = \ddot{x}$$

$$9.8 - 9.8 - 49x = \ddot{x}$$

hence  $\ddot{x} = -49x$  so P moves with S.H.M.

$$c) \underline{x = -0.2} : -0.2 = 0.4 \cos(7t)$$

$$-\frac{1}{2} = \cos 7t \quad (\omega = \sqrt{49} = 7)$$

$$t = \frac{1}{7} \cos^{-1}\left(-\frac{1}{2}\right) = \text{time from C to slack point}$$

$$\dot{x} = \left| -(0.4)(7) \sin(\cos^{-1}(-\frac{1}{2})) \right| \approx \boxed{2.4 \text{ ms}^{-1}}$$

$$\underline{\text{alt}}: v^2 = \omega^2 (a^2 - x^2)$$

$$= 49(0.4^2 - 0.2^2) = \boxed{2.4 \text{ ms}^{-1}}$$

d) when string is slack, P moves under gravity.

$$\begin{array}{l}
 \uparrow \\
 s = \\
 u = 2.4 \\
 v = 0 \\
 a = -g \\
 t = t
 \end{array}
 \left. \vphantom{\begin{array}{l} \uparrow \\ s = \\ u = 2.4 \\ v = 0 \\ a = -g \\ t = t \end{array}} \right\}
 \begin{array}{l}
 v = u + at \\
 0 = 2.4 - gt \\
 t = \frac{2.4}{g} = \text{time from slack to D} \\
 \underline{\underline{g}} = \text{time from D to slack}
 \end{array}$$

but time from C to slack point is equal to the time from slack point to D.

$$= \frac{1}{7} \cos^{-1}\left(-\frac{1}{2}\right) \underline{\underline{=}} \quad (\text{from C})$$

$$\text{So total time} = \frac{1}{7} \cos^{-1}\left(-\frac{1}{2}\right) + \frac{2.4}{g}$$

$$\approx \boxed{0.547 \text{ s}}$$