

M3 June 2017 (MA)

$$Q1) \text{ Area} = \int_0^2 (4 - x^2) dx \quad (\text{at } y=0, x=2)$$

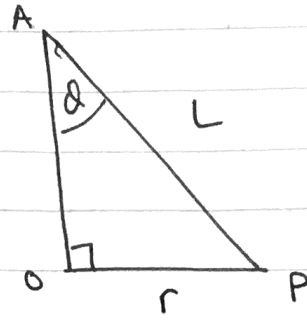
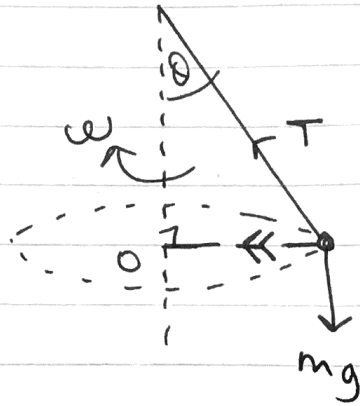
$$= \left[ 4x - \frac{x^3}{3} \right]_0^2 = \left[ 8 - \frac{8}{3} \right] - [0] = \boxed{\frac{16}{3}}$$

$$\text{So mass} = M = \frac{16p}{3} \quad (\text{where } \frac{\text{mass}}{\text{area}} = p)$$

$$M\bar{x} = p \int_0^2 (xy) dx = p \int_0^2 [4x - x^3] dx = p \left[ 2x^2 - \frac{x^4}{4} \right]_0^2$$

$$= p \left[ 2(4) - \frac{2^4}{4} \right] - [0] = \boxed{4p}$$

$$\text{So } \bar{x} = \frac{M\bar{x}}{M} = \frac{4p}{\frac{16p}{3}} = \boxed{\frac{3}{4}}$$

Q2: i)  
/ii

l = length of string.

$$R (\updownarrow) : T \cos \theta = mg$$

$$1.2mg \cos \theta = mg$$

$$\therefore \cos \theta = \frac{1}{1.2} = \frac{5}{6}$$

$$(\rightleftarrows) \underline{N2L(P)} : T \sin \theta = m r \omega^2$$

$$1.2mg \sin \theta = m r (58.8)^2$$

$$\cos \theta = \frac{5}{6} \text{ so } \sin \theta = \sqrt{1 - \left(\frac{5}{6}\right)^2} = \frac{\sqrt{11}}{6}$$

$$\Rightarrow \frac{1.2g\sqrt{11}}{6} = 58.8r$$

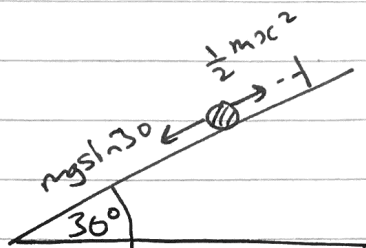
$$\Rightarrow r = \frac{1.2g\sqrt{11}}{6 \times 58.8} = \underline{\underline{0.111m}}$$

$$\text{so } l \sin \theta = 0.111 \dots$$

$$l = \frac{0.111 \dots}{\frac{\sqrt{11}}{6}} = \boxed{0.2m}$$

$$\text{and } \theta = \cos^{-1} \frac{5}{6} = \boxed{33.6^\circ}$$

Q3a)



$$\text{N2L (P)} : \frac{mg}{2} - \frac{1}{2} m v^2 = ma$$

$$\frac{mg}{2} - \frac{m v^2}{2} = m v \frac{dv}{dx}$$

$$g - v^2 = 2v \frac{dv}{dx}$$

$$\int (g - v^2) dx = 2 \int (v) dv$$

$$v^2 = gx - \frac{2x^3}{3} + c$$

$$\underline{x=0, v=0} : 0 = 0 + c$$

$$\therefore c = 0$$

$$\text{so } v^2 = gx - \frac{2x^3}{3}$$

$$\underline{x=3} : v^2 = 3g - \frac{27}{3} = \frac{102}{5}$$

$$v = \sqrt{\frac{102}{5}} = \boxed{4.52 \text{ m/s}}$$

b)  $v^2 = gx - \frac{x^3}{3}$

$v=0$  :  $gx - \frac{x^3}{3} = 0$


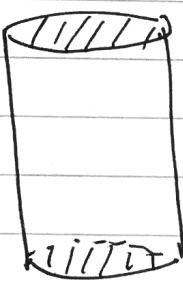
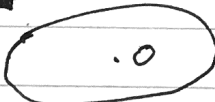

$x \left( g - \frac{x^2}{3} \right) = 0$

$x \neq 0$   
 [when P comes  
 to rest again]

$\therefore g = \frac{x^2}{3}$

$x^2 = 3g$

$\therefore x = \sqrt{3g} \approx \boxed{5.4m}$

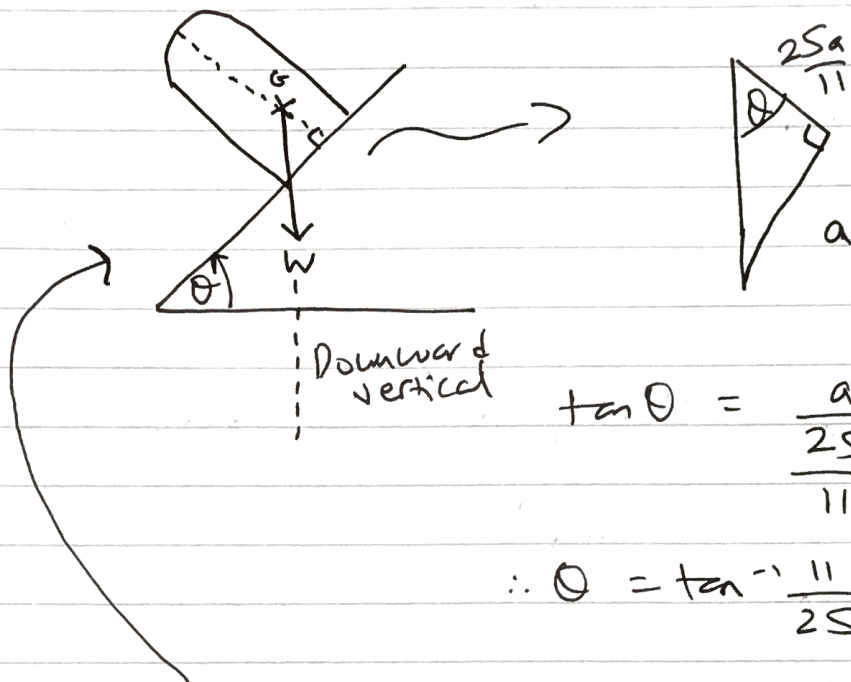
Q40)	Shape	Mass (surface area)	Distance of c.o.m from O
+		$\boxed{2\pi a^2}$	$\boxed{\frac{9a}{2}}$
+		$2\pi r h$ $= \boxed{8\pi a^2}$	$\boxed{2a}$
+		$\boxed{\pi a^2}$	$\boxed{0}$
=		$(8+2+1)\pi a^2$ $= \boxed{11\pi a^2}$	$\boxed{\bar{y}}$

taking moments about diameter of base...

$$2\pi a^2 \left( \frac{9a}{2} \right) + 8\pi a^2 (2a) + \pi a^2 (0) = 11\pi a^2 (\bar{y})$$

$$\frac{9a + 16a}{11} = \bar{y} = \boxed{\frac{25a}{11}}$$

b)



$$\tan \theta = \frac{a}{\frac{25a}{11}} = \frac{11}{25}$$

$$\therefore \theta = \tan^{-1} \frac{11}{25} = \boxed{23.7^\circ}$$

on the point of toppling  $\therefore$  downward vertical passes through lowest point of contact with the plane

Q5a) at A :  $KE = \frac{1}{2} m (7ag)$

$$GPE = mga$$

at B angle  $\theta$  to horizontal :  $KE = \frac{1}{2} mv^2$

$$GPE = mga(1 - \sin\theta)$$

(Lowest point = 0 GPE level)

C.O.E :  $\frac{7amg}{2} + amg = \frac{mv^2}{2} + amg - amg\sin\theta$

$$mv^2 = 7amg + 2amg\sin\theta$$

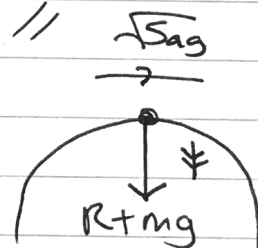
$$\div m : v^2 = 7ag + 2ag\sin\theta$$

$$v^2 = ag(7 + 2\sin\theta)$$

b) at top,  $\theta = 270^\circ$  :  $v^2 = ag(7 + 2\sin 270)$

$$v^2 = 5ag$$

$\downarrow$  N2L (P at top) :  $R + mg = \frac{mv^2}{a}$



$$\Rightarrow R + mg = \frac{m}{a} (5ag)$$

$$\Rightarrow R + mg = 5mg$$

$$\Rightarrow R = 4mg \quad \text{// so } R > 0 \text{ at top hence P remains in contact with the surface..}$$

so P will move in a complete circle.

c) V will be max at the lowest point since this is when GPE is minimum.

$$\text{so } \underline{\text{at } \theta = 90^\circ} : v^2 = 9ag$$

$$\therefore v_{\max} = \boxed{3\sqrt{ag}}$$

$$\underline{\text{alt}} : v^2 = ag(7 + 2\sin\theta)$$

so v will be max when  $\sin\theta = 1$ .

$$\text{hence } v_{\max}^2 = ag(7+2)$$

$$= 9ag$$

$$(\text{so } v = 3\sqrt{ag})$$

$$(Q6a) \quad T = \frac{\lambda c}{c}$$

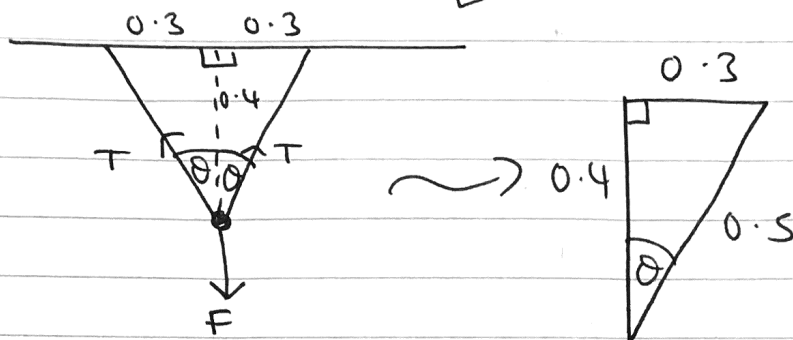
$$8 = \frac{\lambda(0.2)}{0.4}$$

$$8 = \frac{\lambda}{2}$$

$$\times 2: \lambda = 16 \text{ (N)}$$

[no weight acts parallel to this horizontal plane]

b)



$$\cos \theta = \frac{4}{5}$$

↑  $\Sigma F (P) : 2T \cos \theta - F = 0$

$$2T \cos \theta = F$$

$$T = \frac{\lambda x}{L} = \frac{16}{0.2} (0.5 - 0.2) = 24 \text{ N}$$

for one half of the string

so  $2(24) \left(\frac{4}{5}\right) = F = \boxed{38.4 \text{ N}}$

c) using energy : initially :  $KE = 0$

$$EPE = \frac{16}{0.8} (1 - 0.4)^2$$

at AB :  $KE = 0.15 v^2$

$$EPE = \frac{\lambda x^2}{2L} = 8 \times \left(\frac{0.2}{2}\right)^2 = 0.8 \text{ N}$$

C.O.E :  $7.2 = 0.15 v^2 + 0.8$

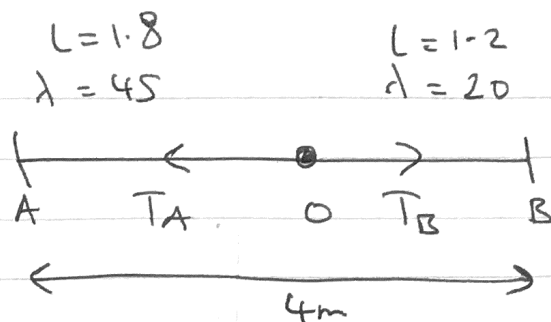
$$v^2 = \frac{7.2 - 0.8}{0.15} = \frac{128}{3}$$

remember there is no change in GPE as P is moving only in the horizontal plane

$$v = \sqrt{\frac{128}{3}} = \boxed{6.53 \text{ m/s}}$$



(Q7a)



$$T_A = T_B$$

$$\frac{45}{1.8} (AO - 1.8) = \frac{20}{1.2} (4 - AO - 1.2)$$

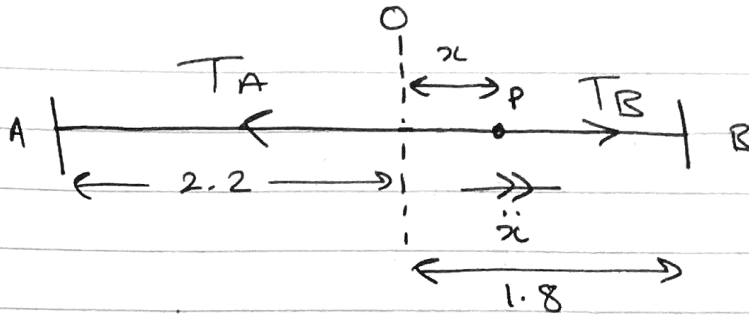
$$25AO - 45 = \frac{50}{3} (2.8 - AO)$$

$$25AO - 45 = \frac{140}{3} - \frac{50}{3}AO$$

$$AO \left( 25 + \frac{50}{3} \right) = \frac{140}{3} + 45$$

$$AO = \frac{\frac{140}{3} + 45}{\frac{50}{3} + 25} = \boxed{2.2\text{m}}$$

b)



$$\text{N2L(P)}: T_B - T_A = 0.6 \ddot{x}$$

$$T_B = \frac{20}{1.2} (1.8 - x - 1.2)$$

$$T_A = \frac{45}{1.8} (2.2 + x - 1.8)$$

$$T_B = 10 - \frac{50}{3} x$$

$$T_A = 10 + 25x$$

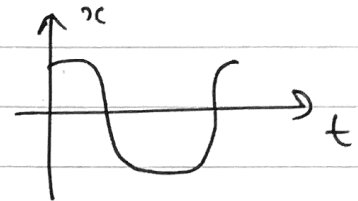
$$\therefore T_B - T_A = 10 - 10 - \frac{125}{3} x =$$

$$\therefore -\frac{125}{3} x = 0.6 \ddot{x}$$

$$\Rightarrow \ddot{x} = -\frac{625}{9} x$$

finding where  $\ddot{x}$  is positive

P starts at an endpoint (C). so  $x = a \cos \omega t$  applies.

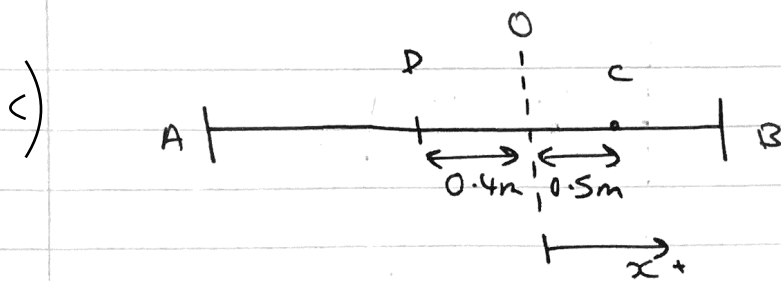


as seen on the graph  $x$  is max at  $t=0$ .

so this means that  $x$  must be increasing in the direction OB.

so  $\ddot{x}$  is also increasing in the direction OB. so  $\ddot{x}$  is positive in this direction.

hence P moves with S.H.M centre O.



$$\ddot{x} = -\frac{625}{9}x$$

$$\therefore \omega = \sqrt{\frac{625}{9}} = \frac{25}{3}$$

and  $a = 0.5$  as  $OC = 0.5m$

$$\therefore x = 0.5 \cos\left(\frac{25t}{3}\right)$$

at D,  $x = -0.4$

$$-0.4 = 0.5 \cos\left(\frac{25t}{3}\right)$$

$$-\frac{4}{5} = \cos\left(\frac{25t}{3}\right)$$

$$\frac{25t}{3} = \cos^{-1}\left(-\frac{4}{5}\right)$$

$$\therefore t = \frac{3}{25} \cos^{-1}\left(-\frac{4}{5}\right)$$

= time from C to D.

from D to A: P will be influenced by  $T_A$  until  $AP = 1.8m$  where AP will become slack.

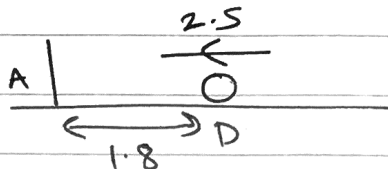
But at D,  $AP = 1.8m$  so from D to A, P will travel at constant speed.

We need to find this 'constant speed'.

recall  $x = 0.5 \cos\left(\frac{25t}{3}\right)$

then  $\ddot{x} = -\frac{25}{6} \sin\left(\frac{25t}{3}\right)$

at  $t = \frac{3}{25} \cos^{-1}\left(-\frac{4}{5}\right) : |\ddot{x}| = 2.5 \text{ ms}^{-1} //$



$$s = vt$$

$$1.8 = 2.5t$$

$$\therefore t = \frac{1.8}{2.5} = \text{time from D to A.}$$

hence total time from C to A =  $\frac{3}{25} \cos^{-1}\left(-\frac{4}{5}\right) + \frac{1.8}{2.5}$

$$= 1.01977\dots \text{s}$$

$$= \boxed{1.02} \text{ sec}$$