

M3 June 2014 (R) (MA)

$$(1a) \quad \frac{dv}{dx} = 3$$

$$\int (1) dv = 3 \int (1) dx$$

$$v = 3x + c$$

$$\underline{v=3, x=2} : 3 = 6 + c$$

$$c = -3 //$$

$$\therefore v = 3x - 3$$

$$\text{at } x=5, v = 3(5) - 3 = 12 \text{ms}^{-1} //$$

$$\therefore v \frac{dv}{dx} = a = 12(3) = 36 \text{ms}^{-2} \text{ at } x=5 //$$

$$F = ma = 0.25 \times 36 = \boxed{9\text{N}}$$

$$b) \quad v = 3x - 3$$

$$\frac{dx}{dt} = 3x - 3$$

$$\left(\frac{1}{x-1}\right) \frac{dx}{dt} = 3$$

$$\int \frac{1}{x-1} dx = \int 3 dt$$

$$\therefore \ln(x-1) = 3t + c //$$

$$x=2, t=0 : \ln(1) = 0 + c$$

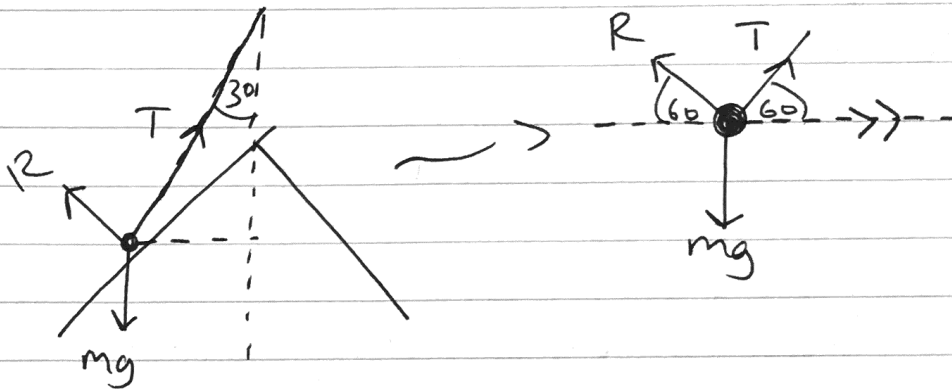
$$c = 0 //$$

$$\text{so } \ln(x-1) = 3t$$

$$x=5 : \ln 4 = 3t$$

$$\therefore t = \frac{1}{3} \ln 4 \approx 0.462$$

Q2a)



$$\text{N2L (particle)} : T \cos 60 - R \cos 60 = m(l \sin 30) \omega^2$$

$$\frac{T-R}{2} = \frac{m l \omega^2}{2}$$

$$\therefore T - R = m l \omega^2 \quad \text{--- (1)}$$

$$R(\uparrow) : T \sin 60 + R \sin 60 = m g$$

$$T + R = \frac{m g}{\sin 60}$$

$$R = \frac{2 m g \sqrt{3}}{3} - T$$

$$\underline{\text{sub R into ①}}: T - \frac{2mg\sqrt{3}}{3} + T = m\omega^2$$

$$2T = m\left(\omega^2 + \frac{2g\sqrt{3}}{3}\right)$$

$$\therefore T = \frac{m}{2}\left(\omega^2 + \frac{2g\sqrt{3}}{3}\right)$$

b) particle remains on the surface so $R > 0$.

$$R = \frac{2mg\sqrt{3}}{3} - T$$

$$R = \frac{2mg\sqrt{3}}{3} - \frac{m\omega^2}{2} - \frac{mg\sqrt{3}}{3}$$

$$R = \frac{mg\sqrt{3}}{3} - \frac{m\omega^2}{2}$$

$$\underline{R > 0}: \frac{mg\sqrt{3}}{3} - \frac{m\omega^2}{2} > 0$$

$$\frac{\omega^2}{2} < \frac{g\sqrt{3}}{3}$$

$$\omega^2 < \frac{2g\sqrt{3}}{3L} //$$

$$\therefore \omega < \sqrt{\frac{2g\sqrt{3}}{3L}}$$

$$T = \frac{2\pi}{\omega}$$

$$\frac{2\pi}{T} = \omega$$

$$\text{so } \frac{2\pi}{T} < \sqrt{\frac{2g\sqrt{3}}{3L}}$$

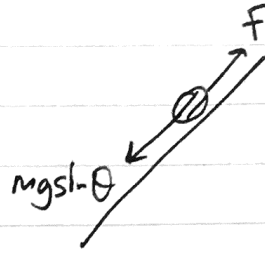
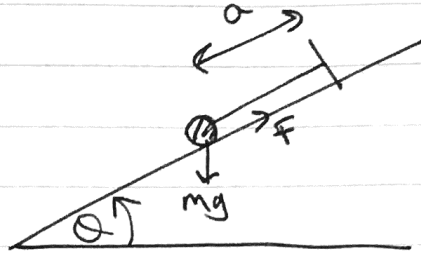
take reciprocal of both sides so signs flip:

$$\frac{T}{2\pi} > \sqrt{\frac{3L}{2g\sqrt{3}}}$$

either form is ok.

$$T > 2\pi \sqrt{\frac{3L}{2g\sqrt{3}}} \rightarrow T > 2\pi \sqrt{\frac{L\sqrt{3}}{2g}}$$

Q3a)



Initially :

$$GPE = mgd \sin \theta$$

$$KE = 0$$

$$EPE = 0$$

let distance moved down
the plane before
coming to rest = d

Finally :

$$KE = 0$$

$$GPE = 0$$

$$EPE = \frac{mg}{2a} (d)^2$$

$$+ W \cdot D \text{ by friction} = F \times d = \mu R \times d = (mg \cos \theta) d \cdot \mu$$

C.O.E

$$\Rightarrow mgd \sin \theta = \frac{mgd^2}{2a} + mgd \mu \cos \theta$$

total energy
initially

total energy
finally

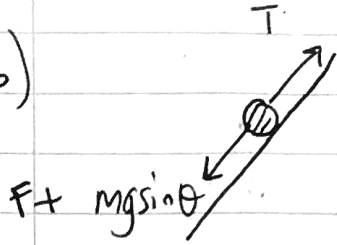
work done
by friction

$$\Rightarrow \underline{mgd} : \sin \theta = \frac{d}{2a} + \mu \cos \theta$$

$$\times 2a : 2a \sin \theta - 2a \mu \cos \theta = d$$

$$\therefore d = 2a (\sin \theta - \mu \cos \theta)$$

b)



$$R(\uparrow): F + mg \sin \theta = T$$

$$F + mg \sin \theta = \frac{mg}{a} (d)$$

$$F = \frac{mg}{a} (2a)(\sin \theta - \mu \cos \theta) - mg \sin \theta$$

$$F = 2mg \sin \theta - 2mg \mu \cos \theta - mg \sin \theta$$

$$F = mg \sin \theta - 2mg \mu \cos \theta$$

$$\text{but } F \leq \mu R.$$

$$R = mg \cos \theta$$

$$F \leq \mu mg \cos \theta$$

$$\text{and } F = mg \sin \theta - 2mg \mu \cos \theta$$

$$\therefore mg \sin \theta - mg \mu \cos \theta \leq 2\mu mg \cos \theta$$

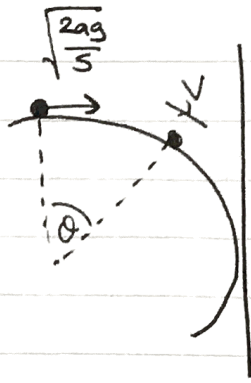
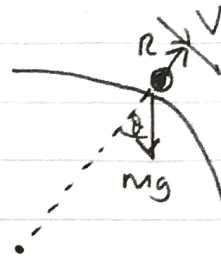
$$mg \sin \theta \leq 3\mu mg \cos \theta$$

$$\sin \theta \leq 3\mu \cos \theta$$

$$\text{hence } \mu \geq \frac{\sin \theta}{3 \cos \theta}$$

$$\Rightarrow \mu \geq \frac{1}{3} \tan \theta$$

(Q4a)

when the particle leaves the sphere

$$+ \sqrt{N \leq L} \text{ (particle)} : mg \cos \theta - R = \frac{mv^2}{a}$$

$$\Rightarrow \underline{R=0} : mg \cos \theta = \frac{mv^2}{a}$$

$$\Rightarrow \underline{v^2 = ag \cos \theta}$$

Energy from top to when particle leaves sphere:

$$\text{At top : KE} = \frac{1}{2} m \left(\frac{2ag}{5} \right) = \frac{2amg}{10} = \frac{amg}{5}$$

$$\text{GPE} = mga$$

$$\text{At point where particle leaves the sphere : KE} = \frac{1}{2} mv^2$$

$$\text{GPE} = mg a \cos \theta$$

$$\text{C.O.E : } \frac{amg}{5} + amg = \frac{mv^2}{2} + amg \cos \theta$$

$$\frac{6}{5} ag - ag \cos \theta = \frac{v^2}{2}$$

$$v^2 = \frac{12}{5} ag - 2 ag \cos \theta$$

$$\text{but } \underline{v^2 = ag \cos \theta} : ag \cos \theta = \frac{12ag}{5} - 2 ag \cos \theta$$

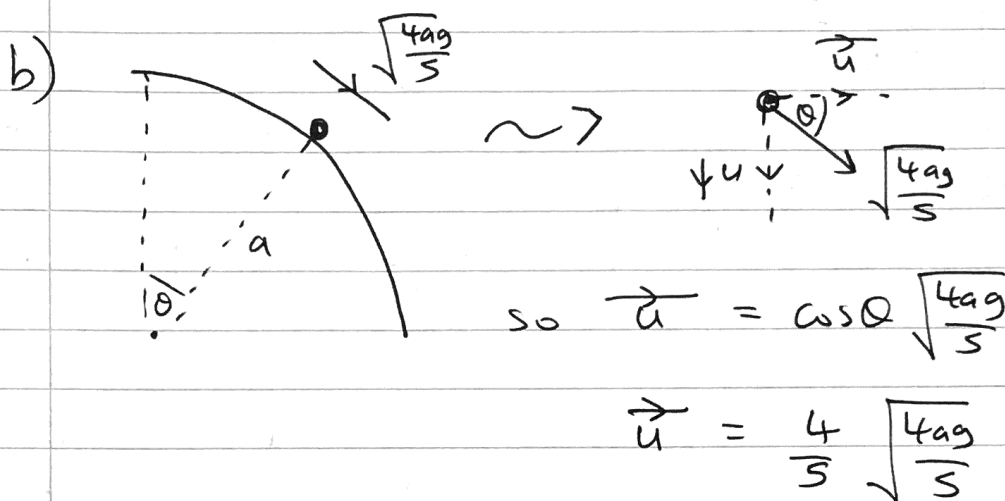
$$3 ag \cos \theta = \frac{12ag}{5}$$

$$3 \cos \theta = \frac{12}{5}$$

$$\therefore \cos \theta = \frac{12}{15} = \frac{4}{5} //$$

$$\text{so } v^2 = ag \left(\frac{4}{5} \right)$$

$$\therefore v = \sqrt{\frac{4ag}{5}}$$



$$\text{and } u \sin \theta = \sqrt{\frac{4ag}{5}} \sin \theta = \frac{3}{5} \sqrt{\frac{4ag}{5}}$$

$$\begin{aligned} \text{distance from wall initially} &= a - a \sin \theta \\ &= a(1 - \sin \theta) \\ &= a \left(1 - \frac{3}{5} \right) = \frac{2a}{5} // \end{aligned}$$

consider horizontal motion: $s = ut$.

$$\Rightarrow \frac{2a}{5} = \left(\frac{4}{5} \sqrt{\frac{4ag}{5}} \right) t$$

$$2a = \left(4 \sqrt{\frac{4ag}{5}}\right) t$$

$$\frac{a}{2 \sqrt{\frac{4ag}{5}}} = t = \frac{a}{2} \sqrt{\frac{5}{4ag}} = \frac{1}{2} \sqrt{\frac{5a}{4g}} =$$

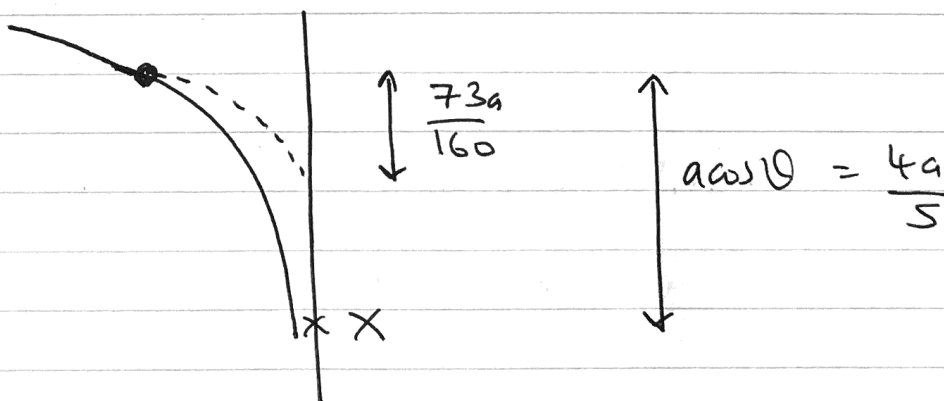
consider vertical motion:

$$\left. \begin{array}{l} s = x \\ u = \frac{3}{5} \sqrt{\frac{4ag}{5}} \\ v = \\ a = g \\ t = \frac{1}{2} \sqrt{\frac{5a}{4g}} \end{array} \right\} \begin{array}{l} s = ut + \frac{1}{2} at^2 \\ x = \frac{3}{10} \sqrt{\frac{4ag}{5}} \sqrt{\frac{5a}{4g}} + \frac{g}{2} \left(\frac{1}{4}\right) \left(\frac{5a}{4g}\right) \\ x = \frac{3}{10} \sqrt{a^2} + \frac{5a}{32} \end{array}$$

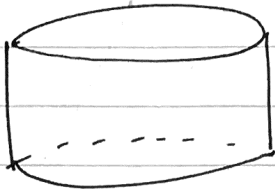

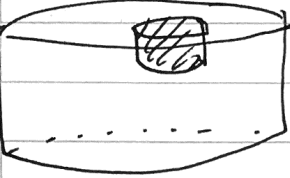
$$x = \frac{73}{160} a = \text{distance travelled.}$$

$$\text{so } AX = \frac{4a}{5} - \frac{73}{160} a = \boxed{\frac{11a}{32}}$$

$a \cos \theta$



● (Q5a)

Shape	Mass (vol.)	Distance of c.o.m from O
	$\pi r^2 h$	$\frac{h}{2}$
	$\pi \left(\frac{1}{4}r\right)^2 \left(\frac{h}{4}\right)$ $= \frac{\pi r^2 h}{64}$	$\frac{h}{8}$
	$\frac{63\pi r^2 h}{64}$	\bar{y}

taking moments about a diameter of the plane face containing O...

$$\pi r^2 h \left(\frac{h}{2}\right) - \frac{\pi r^2 h}{64} \left(\frac{h}{8}\right) = \frac{63\pi r^2 h}{64} (\bar{y})$$

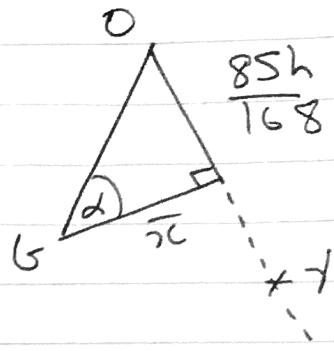
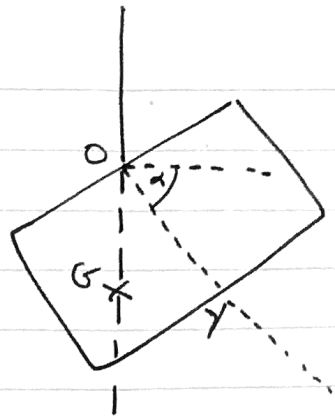
$$\frac{\frac{h}{2} - \frac{h}{8(64)}}{\frac{63}{64}} = \bar{y} = \frac{\frac{255h}{512}}{\frac{63}{64}}$$

$$= \frac{85h}{168}$$

We are told that

$$\tan d = 17$$

b)



Downward vertical

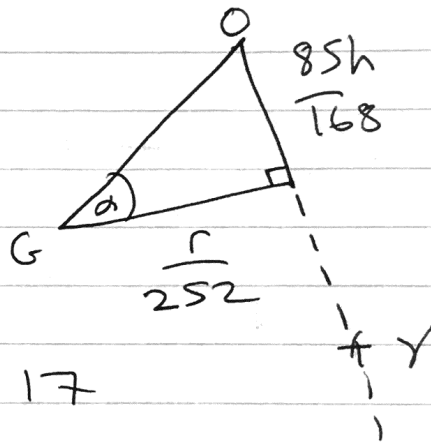
to find \bar{x} now,

Moments about O, but use displacement of c.o.m from O of this time

$$\pi r^2 h (0) - \frac{\pi r^2 h}{64} \left(\frac{r}{4}\right) = \frac{63 \pi r^2 h}{64} (\bar{x})$$

$$\bar{x} = \frac{-\frac{r}{64(4)}}{\frac{63}{64}} = -\frac{r}{252}$$

So now:



$$\tan d = \frac{85h}{168} = 17$$

$$\frac{17r}{252} = \frac{85h}{168}$$

$$r = \frac{85h}{168} \times \frac{252}{17} = \frac{15h}{2}$$

$$\therefore \boxed{r = \frac{15h}{2}}$$

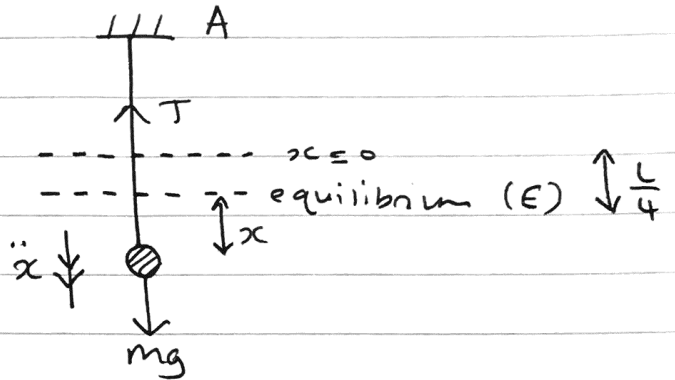
● (Q6a) $T = mg = \frac{\lambda x}{L}$
 $\frac{4mg}{L} (e) = mg$
 $\Rightarrow \frac{4e}{L} = 1$



$\Rightarrow \boxed{e = \frac{L}{4}}$

● b) $mg - T = m \ddot{x} \left[\downarrow_{+} (N2L(P)) \right]$

$T = \frac{4mg}{L} \left(\frac{L}{4} + x \right)$

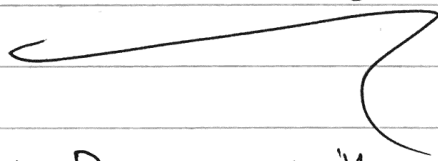


$\therefore mg - \frac{4mgL}{4L} - \frac{4mgx}{L} = m \ddot{x}$

$\Rightarrow mg - mg - \frac{4mgx}{L} = m \ddot{x}$

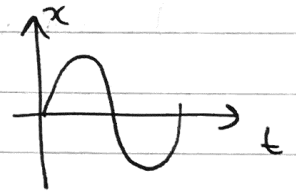
$\Rightarrow -\frac{4mgx}{L} = m \ddot{x}$

$\Rightarrow \ddot{x} = -\frac{4gx}{L}$



$\therefore P$ moves with S.H.M.

● P starts at equilibrium position so $x = 0$ as $t = 0$ applies.



● so the initial velocity of P is positive (as seen from the graph). hence x increases in the direction AE. so \ddot{x} also increases in this direction so \ddot{x} is positive \downarrow .

$$c) \quad \ddot{x} = -\frac{4g}{L} x$$

$$\omega = \sqrt{\frac{4g}{L}}$$

speed of projection = max speed
(as this is at the centre of the oscillation)

$$\therefore v_{\max} = \sqrt{gL} = a\omega$$

$$\sqrt{gL} = a \sqrt{\frac{4g}{L}}$$

$$\sqrt{L} = a \sqrt{\frac{4}{L}}$$

$$L = a\sqrt{4}$$

$$\boxed{a = \frac{L}{2}}$$

d) When string first becomes slack, $x = -\frac{L}{4}$

($x = -\frac{L}{4}$ as above the equilibrium point x is negative)

$$x = a \sin \omega t :$$

$$-\frac{L}{4} = \frac{L}{2} \sin \left(t \sqrt{\frac{4g}{L}} \right)$$

$$-\frac{1}{2} = \sin \left(t \sqrt{\frac{4g}{L}} \right)$$

$$\sin^{-1} \left(-\frac{1}{2} \right) = -\frac{\pi}{6}, \frac{7\pi}{6}$$

$$t \sqrt{\frac{4g}{L}} = \sin^{-1} \left(-\frac{1}{2} \right) = -\frac{\pi}{6}, \frac{7\pi}{6}$$

$$t > 0 \quad \therefore \frac{7\pi}{6} = t \sqrt{\frac{4g}{L}}$$

$$\text{So } t = \frac{7\pi}{6} \sqrt{\frac{L}{4g}} = \frac{7\pi}{12} \sqrt{\frac{L}{2g}}$$
