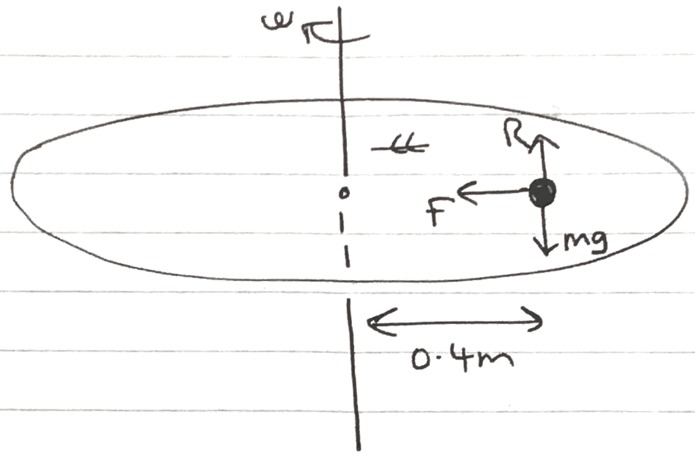
M3 June 2013 (MA)

Q1)



20 revolutions in 60s.

$\therefore$ 1 revolution occurs in $\frac{60}{20} = 3s$

so $T = 3$

$\therefore \omega = \frac{2\pi}{3}$

$R(\updownarrow)$: $R = mg$

N2L(P) : $F = mr\omega^2$

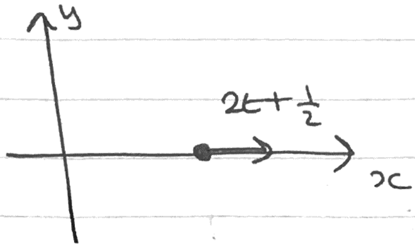
$F = m(0.4)\left(\frac{2\pi}{3}\right)^2$

P is on the point of slipping $\therefore F = \mu R$

$\Rightarrow 0.4\cancel{m}\left(\frac{2\pi}{3}\right)^2 = \mu(\cancel{m}g)$

$\Rightarrow \mu = \frac{0.4\left(\frac{2\pi}{3}\right)^2}{g} = \boxed{0.179}$

Q2a)



$$F = 2t + \frac{1}{2} = 0.5a$$

$$a = \frac{dv}{dt} \quad : \quad 2t + \frac{1}{2} = 0.5\frac{dv}{dt}$$

$$\Rightarrow \quad 4t + 1 = \frac{dv}{dt}$$

$$\Rightarrow \quad \int (1)\,dv = \int (4t+1)\,dt$$

$$\Rightarrow \quad v = 2t^2 + t + c$$

$\underline{t=0, v=0}$: $0 = 0 + 0 + c$

$\therefore \underline{\underline{c = 0}}$

so $\boxed{v = 2t^2 + t}$

b) $x = \int (v)\,dt \longrightarrow x = \int (2t^2 + t)\,dt$

$$x = \left[\frac{2t^3}{3} + \frac{t^2}{2}\right] + c$$

$$x = \frac{2}{3}t^3 + \frac{t^2}{2} + c$$

$\underline{t=0, x=0}$: $0 = c$ //

so $x = \frac{2}{3}t^3 + \frac{t^2}{2}$

from (a), $v = 2t^2 + t$

at $v = 6$: $6 = 2t^2 + t$

$$2t^2 + t - 6 = 0$$

$$(2t - 3)(t + 2) = 0$$

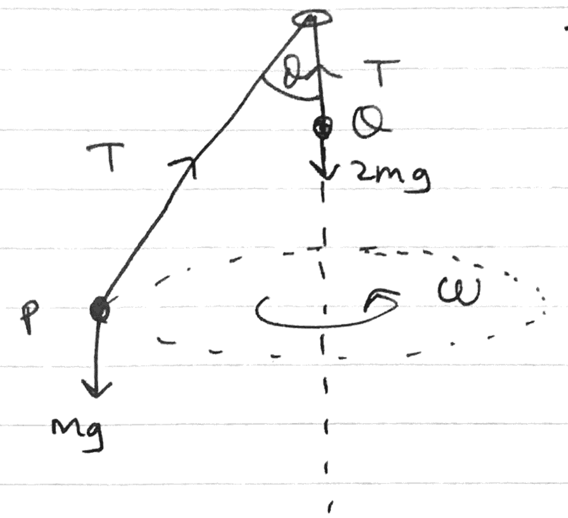
$$t = \frac{3}{2}, \quad t \neq -2 \quad (t \geq 0)$$

So $v = 6$ at $t = \frac{3}{2}$.

Now substitute $t = \frac{3}{2}$ into $\left[x = \frac{2t^3}{3} + \frac{t^2}{2}\right]$ to find distance OA.

$t = \frac{3}{2}$: $OA = \frac{2}{3}\left(\frac{3}{2}\right)^3 + \frac{\left(\frac{3}{2}\right)^2}{2} = \boxed{\frac{27}{8}}$

Q3i/ii)



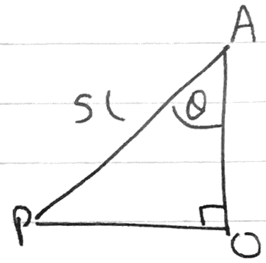
Tension is the same for both particles as they are connected to the same light inextensible string passing through the ring.

For Q : $R(\updownarrow)$: $T = 2mg$ //

For P : $R(\updownarrow)$: $T\cos\theta = mg$

$(T = 2mg) \Rightarrow \quad 2mg\cos\theta = mg$

$\therefore \cos\theta = \frac{1}{2}$ // $\rightarrow$



(Length AP $= 6l - 1 = 5l$)

$\theta = \cos^{-1}\frac{1}{2} = 60°$

So $\cos\theta = \frac{AO}{5l} = \frac{1}{2}$

$\therefore AO = \frac{5l}{2}$ //

$\therefore PO = \text{radius} = \sqrt{(5l)^2 - (\frac{5l}{2})^2} = \frac{5l\sqrt{3}}{2}$ //

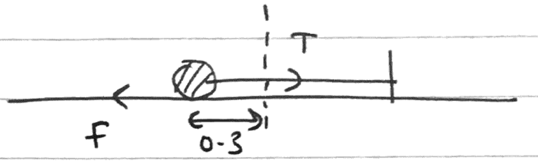
N2L (P) $\xrightarrow{+}$: $T\sin\theta = mr\omega^2$

$2\cancel{m}g\sin 60 = \cancel{m}\left(\frac{5l\sqrt{3}}{2}\right)\omega^2$

$\frac{2g\sin 60 \times 2}{5l\sqrt{3}} = \omega^2 = \frac{2\sqrt{3}\,g}{5l\sqrt{3}} = \frac{2g}{5l}$ //

$$\omega^2 = \left(\frac{2g}{5l}\right) \quad \therefore \omega = \sqrt{\frac{2g}{5l}}$$

Q4a)



at OP = 1·2m, the string becomes slack.

<u>considering energy</u>

<u>At B</u> : $KE = 0$

$$EPE = \frac{\lambda x^2}{2l} = \frac{\lambda x}{l} \times \frac{x}{2} = 20 \times \frac{0\cdot3}{2} = 3N$$

Tension = 20N at B

<u>At OP=1·2m</u> : $KE = \frac{mv^2}{2}$

$$EPE = 0 \quad (x=0).$$

and work done by friction = $\mu Rs = \frac{2}{5}(2g)(0\cdot3)$

[R = 2g (by resolving perpendicular to plane)]

$\Rightarrow$ <u>By C.O.E</u> : $\underbrace{3}_{\text{Total energy at start}} = \underbrace{\frac{mv^2}{2} + \frac{4g}{5}(0\cdot3)}_{\text{Total energy at end + W.D by friction}}$

$$3 = v^2 + 2.352$$

$$v^2 = \frac{81}{125} \quad \therefore v = \sqrt{\frac{81}{125}} = \boxed{0.80\ m/s}$$

b) Using energy again, total energy initially = KE

$$= \frac{1}{2}(2)(0.80)^2$$

total KE initially = Work done by friction

(from OP = 1.2 to rest)

$$\therefore \frac{1}{2}(2)(0.80)^2 = \frac{2}{5}(2g)(d)$$

$$\frac{(0.80)^2}{\frac{2}{5}(2g)} = d = 0.083m$$

$$\therefore BC = 0.083 + 0.3 = \boxed{0.38}\ m$$

from (OP = 1.2) to OC   from OB to (OP = 1.2)

Q5a) $V = \pi \int_0^2 y^2 \, dx = \pi \int_0^2 (x+1)^4 \, dx$

$= \pi \left[\frac{(x+1)^5}{5} \right]_0^2 = \pi \left[\frac{3^5}{5} \right] - \pi \left[\frac{1}{5} \right]$

$= \left[\frac{243}{5} - \frac{1}{5} \right] \pi = \frac{242\pi}{5}$

$\therefore \quad M = \frac{242 p\pi}{5}$.

$M\bar{x} = p\pi \int_0^2 (y^2 x) \, dx = p\pi \int_0^2 \left[x (x+1)^4 \right] dx$

$= p\pi \int_0^2 (x+1-1)(x+1)^4 \, dx = p\pi \int_0^2 (x+1)^5 - (x+1)^4 d$

$= p\pi \left[\frac{(x+1)^6}{6} - \frac{(x+1)^5}{5} \right]_0^2 = p\pi \left[\frac{3^6}{6} - \frac{3^5}{5} \right]$

$- p\pi \left[\frac{1}{6} - \frac{1}{5} \right]$

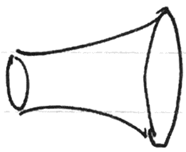
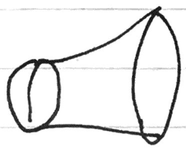
$= \left[\frac{729}{10} + \frac{1}{30} \right] \times p\pi = \frac{1094 p\pi}{15} = M\bar{x}$ //

$\bar{x} = \frac{M\bar{x}}{M} = \frac{\frac{1094 \cancel{p\pi}}{15}}{\frac{242 \cancel{p\pi}}{5}} = \boxed{\frac{547}{363} \simeq 1.51}$

b) Mass per unit volume of hemisphere is 10x that of S.

$\therefore \quad m_{hemisphere} = 10p \times \text{volume}$

$m_S = p \times \text{volume}.$

Shape	Mass (vol.)	Distance of c.o.m from A
	$\boxed{p\left[\frac{242\pi}{5}\right]}$	$2 - \frac{547}{363} = \boxed{\frac{179}{363}}$
	$\boxed{10p\left[\frac{2}{3}\pi(1)^3\right]}$	$2 + \frac{3}{8}(1) = \boxed{\frac{19}{8}}$
	$\left(\frac{242}{5} + \frac{20}{3}\right)p\pi = \boxed{\frac{826p\pi}{15}}$	$\boxed{\bar{x}}$

$y = (x+1)^2$

$x = 0, y = 1$

$\therefore r = 1$

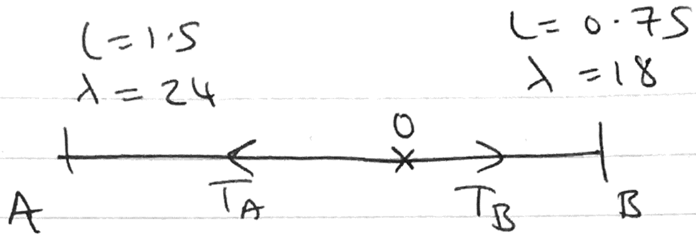
Moments about A . . .

$$\frac{242}{5}\left(\frac{179}{363}\right) + \frac{20}{3}\left(\frac{19}{8}\right) = \frac{826}{15}(\bar{x})$$

$$\bar{x} = \frac{\frac{242}{5}\left(\frac{179}{363}\right) + \frac{20}{3}\left(\frac{19}{8}\right)}{\frac{826}{15}} = \frac{1191}{1652}$$

$$= \boxed{0.72}$$

Q6a)



$$T_A = T_B$$

$$\frac{24}{1.5}(AO-1.5) = \frac{18}{0.75}(3.75 - AO - 0.75)$$

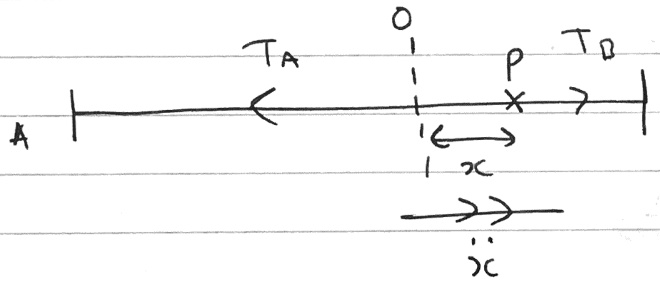
$$16(AO-1.5) = 24(3-AO)$$

$$16AO - 24 = 72 - 24AO$$

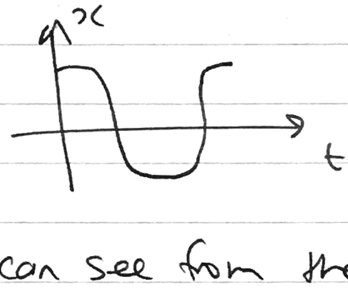
$$40AO = 96$$

$$\therefore AO = \frac{96}{40} = \boxed{2.4m}$$

b)



P starts at an endpoint, so $x = a\cos\omega t$ applies.



$\underline{\overset{\rightarrow +}{N2L}}$ (P): $T_B - T_A = 0.8\ddot{x}$

$$\frac{18}{0.75}(1.35 - x - 0.75) = T_B$$

$$\frac{24}{1.5}(2.4 + x - 1.5) = T_A$$

We can see from the graph that at t=0, x is maximum. So this means that x increases in the direction OB. Hence $\ddot{x}$ is positive in this direction.

$$\therefore 24(0.6 - x) - 16(x + 0.9) = 0.8\ddot{x}$$

$$14.4 - 24x - 16x - 14.4 = 0.8\ddot{x}$$

$$-40x = 0.8\ddot{x}$$

$$-50x = \ddot{x} \qquad \therefore P \text{ moves with S.H.M}$$

c) $\omega = \sqrt{50}$. $V_{max} = a\omega = \sqrt{2}$

$$\therefore \sqrt{2} = a\sqrt{50} \qquad \therefore a = \sqrt{\frac{2}{50}} = \frac{1}{5}$$

$$x = \frac{1}{5}\cos(t\sqrt{50})$$

$$-0.1 = 0.2\cos(t\sqrt{50})$$

$$-\frac{1}{2} = \cos(t\sqrt{50})$$

P will be 0.1m from the equilibrium position and moving in the negative x-direction

$$\cos^{-1}\left(-\frac{1}{2}\right) = t\sqrt{50} = \frac{2\pi}{3}$$

$$\therefore t = \frac{2\pi}{3\sqrt{50}} = \boxed{0.296s}$$

Q7a) At A : $KE = \frac{1}{2}(5m)\left(\frac{9ag}{5}\right) = \frac{9amg}{2}$

$GPE = 5mga$

At angle θ : $KE = \frac{5m}{2} v^2$

$GPE = 5mga(1-\cos\theta)$

C.O.E : $\frac{9amg}{2} + 5amg = \frac{5mv^2}{2} + 5amg(1-\cos\theta)$

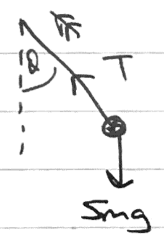
$\frac{9amg}{2} + 5amg - 5amg = \frac{5mv^2}{2} - 5amg\cos\theta$

$\frac{9amg}{2} + 5amg\cos\theta = \frac{5mv^2}{2}$

$9ag + 10ag\cos\theta = 5v^2$

$\therefore v^2 = \frac{1}{5}ag(9 + 10\cos\theta)$

At angle θ...



N2L $\nwarrow +$ (P) : $T - 5mg\cos\theta = \frac{5m}{a} v^2$

$T - 5mg\cos\theta = \frac{5m}{a}\left(\frac{ag}{5}\right)(9 + 10\cos\theta)$

$$T - 5mg\cos\theta = mg(9 + 10\cos\theta)$$

$$T = 9mg + 10mg\cos\theta + 5mg\cos\theta$$

$$T = 9mg + 15mg\cos\theta = 3mg(3 + 5\cos\theta)$$

b) $T = 0$: $3mg(3 + 5\cos\theta) = 0$

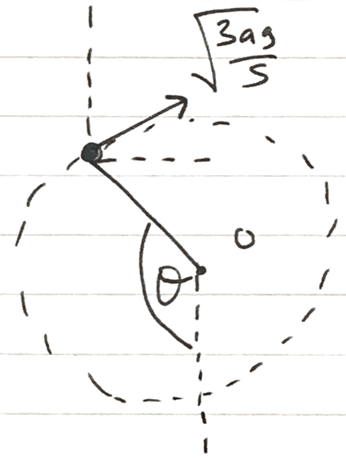
$$5\cos\theta = -3$$

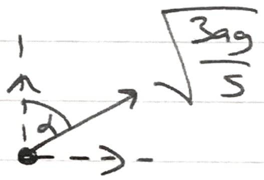
$$\therefore \cos\theta = -\frac{3}{5}$$

$$v^2 = \frac{ag}{5}\left(9 + 10\left(-\frac{3}{5}\right)\right) = \frac{3ag}{5}$$

$$\therefore \boxed{v = \sqrt{\frac{3ag}{5}}}$$

c) i)





$$\cos\theta = -\frac{3}{5}$$

let $\alpha = \theta - 90$,

$$\cos\theta = \cos(90 - \theta) = -\frac{3}{5}$$

α = angle $\vec{P}$ initially makes with the vertical

$$\therefore \sin(-(-90 + \theta)) = -\frac{3}{5}$$

$$\sin(-\alpha) = -\frac{3}{5}$$

$$\therefore \sin\alpha = \frac{3}{5}, \text{ so } \cos\alpha = 4/5$$

so $\overrightarrow{u} = \sqrt{\frac{3ag}{5}} \sin\alpha = \frac{3}{5}\sqrt{\frac{3ag}{5}}$ //

and $u\uparrow = \sqrt{\frac{3ag}{5}} \cos\alpha = \frac{4}{5}\sqrt{\frac{3ag}{5}}$

$s = ut$ for horizontal vel. $\therefore$ $s = \frac{3}{5}t\sqrt{\frac{3ag}{5}}$

initial $\overrightarrow{\text{distance}}$ from $O = a\cos\alpha$

$= \frac{4}{5}a$ //

hence $x = \frac{3}{5}t\sqrt{\frac{3ag}{5}} - \frac{4a}{5}$ //

ii) considering vertical velocity.

$+\uparrow$
$s = s$
$u = \frac{4}{5}\sqrt{\frac{3ag}{5}}$
$v =$
$a = -g$
$t = t$

$s = ut + \frac{1}{2}at^2$

$s = \frac{4}{5}t\sqrt{\frac{3ag}{5}} - \frac{g}{2}t^2$

initial distance $\uparrow$ from $O = a\sin\alpha$

$= \frac{3}{5}a$ //

$\therefore y = \frac{4}{5}t\sqrt{\frac{3ag}{5}} - \frac{g}{2}t^2 + \frac{3a}{5}$