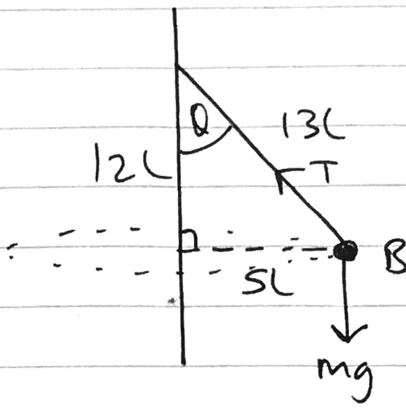
M3 June 2010 (MA)

Q1a)



$\cos\theta = \frac{12}{13}$

$\sin\theta = \frac{5}{13}$

$R(\updownarrow): T\cos\theta = mg$

$$\frac{12}{13}T = mg \quad \therefore \quad T = \boxed{\frac{13mg}{12}}$$

b) $\overset{+}{\leftarrow}$ N2L (B) : $T\sin\theta = \frac{mv^2}{5l}$

$$\frac{13mg}{12}\left(\frac{5}{13}\right)(5l) = mv^2$$

$$\frac{25\cancel{m}gl}{12} = \cancel{m}v^2$$

$$v^2 = \frac{25gl}{12}$$

$$\therefore v = 5\sqrt{\frac{gl}{12}} = \boxed{\frac{5}{2}\sqrt{\frac{gl}{3}}}$$

Q2a) $F \propto \frac{1}{x^2}$: $F = \frac{k}{x^2}$

$\underline{x = R, F = mg}$: $mg = \frac{k}{R^2}$

$\therefore k = mgR^2.$

hence $F = \frac{mgR^2}{x^2}$

b) $-\frac{mgR^2}{x^2} = ma = mv\frac{dv}{dx}$

$-\frac{gR^2}{x^2} = v\frac{dv}{dx}$

$\int (v)\, dv = -gR^2 \int \left(\frac{1}{x^2}\right) dx$

$\frac{v^2}{2} = \frac{+gR^2}{x} + c.$

$\underline{v = 3u, x = R}$: $\frac{9u^2}{2} = \frac{gR^2}{R} + c$

$c = \frac{9u^2}{2} - gR$

$$\therefore \quad \frac{v^2}{2} = \frac{gR^2}{x} + \frac{9u^2}{2} - gR$$

$v = u,\ x = 2R$: $\quad \frac{u^2}{2} = \frac{gR^2}{2R} + \frac{9u^2}{2} - gR$

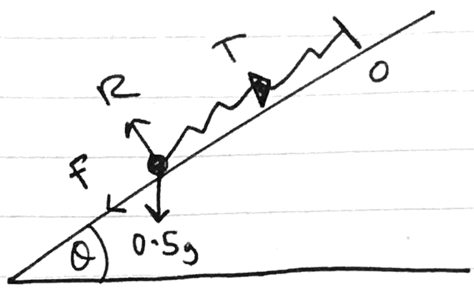
$$\frac{u^2}{2} = -\frac{gR}{2} + \frac{9u^2}{2}$$

$$4u^2 = \frac{gR}{2}$$

$$\therefore \ u^2 = \frac{gR}{8}$$

$$\text{hence} \quad u = \sqrt{\frac{gR}{8}}$$

Q3)



$R\updownarrow$: $R = 0.5g\cos\theta$

$R = \frac{98}{25}$

Initially

$KE = 0$

$EPE = \frac{\lambda(1.5-0.9)^2}{1.8}$

$GPE = 0$

After moving 0.7m up

$KE = 0$

$EPE = \frac{\lambda(0.9-0.8)^2}{2(0.9)}$

$GPE = 0.5g(0.7\sin\theta)$

$+$ W·D by friction $= \underbrace{\left(\frac{98}{25}\right)}_{R}\underbrace{(0.15)}_{\mu}\underbrace{(0.7)}_{s}$

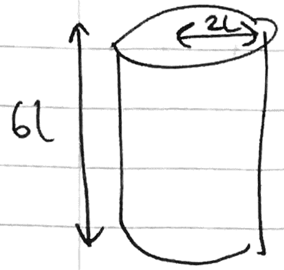
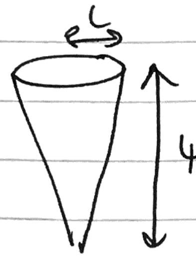
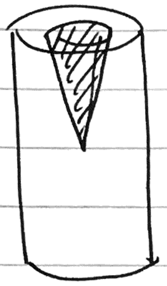
So... : $\underbrace{\frac{\lambda(1.5-0.9)^2}{1.8}}_{\text{Total energy at start}} = \underbrace{\frac{\lambda(0.1)^2}{1.8} + 0.5(0.7)g(\sin\theta) + W\cdot D.}_{\text{Total energy at end + work done}}$

$\Rightarrow \frac{\lambda(0.6)^2}{1.8} - \frac{\lambda(0.1)^2}{1.8} = 0.5(0.7g)\left(\frac{3}{5}\right) + \frac{98}{25}(0.15)(0.7)$

$\Rightarrow \lambda\frac{(0.6^2 - 0.1^2)}{1.8} = \frac{3087}{1250}$

$\Rightarrow \lambda = \frac{\frac{3087}{1250}(1.8)}{0.6^2 - 0.1^2} = \boxed{12.7N}$

Q4a)

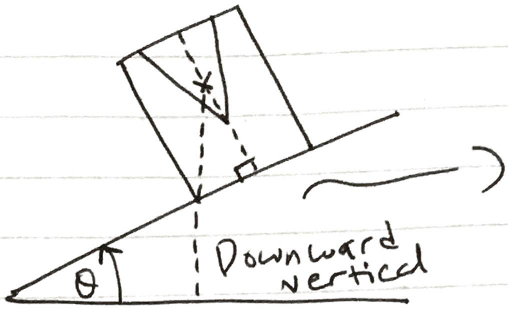
Shape	Mass (vol.)	Distance of c.o.m from O
	$\pi(2l)^2(6l) = \boxed{24\pi l^3}$	$\boxed{3l}$
(−) 	$\frac{1}{3}\pi(l)^2(4l) = \boxed{\frac{4\pi l^3}{3}}$	$\frac{4l}{4} = \boxed{l}$
	$\boxed{\frac{68\pi l^3}{3}}$	$\boxed{\bar{y}}$

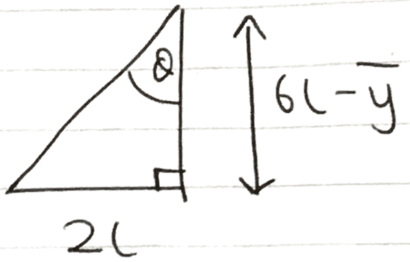
Moments about O...

$$24(3l) - \frac{4}{3}(l) = \frac{68}{3}(\bar{y})$$

$$\frac{\left[24(3) - \frac{4}{3}\right]l}{\frac{68}{3}} = \bar{y} = \boxed{\frac{53l}{17}}$$

b)





On point of toppling ∴ downward vertical passes through lowest point of contact with plane

$6l - \bar{y} = \frac{49l}{17}$ //

$\therefore \tan\theta = \frac{2}{\frac{49}{17}} = \frac{34}{49}$

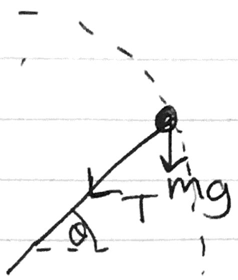
$\therefore \theta = \tan^{-1}\frac{34}{49} = \boxed{34.8^\circ}$

Q5a) using conservation of energy:

$$\underbrace{\frac{mu^2}{2} - \frac{mv^2}{2}}_{\Delta KE} = \underbrace{mga\sin\theta}_{\Delta GPE}$$

$$\therefore \frac{u^2}{2} - ag\sin\theta = \frac{v^2}{2}$$

$$\therefore v^2 = u^2 - 2ag\sin\theta = \boxed{5ag - 2ag\sin\theta}$$

b) 

$\overset{+}{\swarrow}$ N2L (P) : $T + mg\sin\theta = \frac{mv^2}{a}$

$\therefore \; T = \frac{mv^2}{a} - mg\sin\theta$

$\Rightarrow \; T = \frac{m}{a}(5ag - 2ag\sin\theta) - mg\sin\theta$

$\Rightarrow \; T = 5m\cancel{a}g - 2\cancel{a}mg\sin\theta - mg\sin\theta$

$\Rightarrow \; T = 5mg - 3mg\sin\theta$

c) at $\theta = 90°$: $T = 5mg - 3mg$

$T = 2mg > 0$ at the top.

so $T > 0$ at the top hence P moves in complete circles.

d) max speed occurs at the lowest point. (This is where GPE is lowest $\therefore$ KE is max).

$\theta = 270°$: $T = 5mg - 3mg\sin 270$

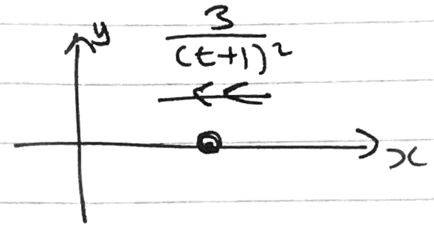
$T = 5mg$

$\theta = 270°$: $v^2 = u^2 - 2ag(\sin 270)$

$v^2 = 5ag + 2ag = 7ag$ //

$\therefore\ V = \boxed{\sqrt{7ag}}$

(6a)



$$a = -\frac{3}{(t+1)^2} = \frac{dv}{dt}$$

$$-3\int \frac{1}{(t+1)^2}\,dt = \int (1)\,dv$$

$$-3\left[-(t+1)^{-1}\right] = v + c$$

$$\therefore\ V + c = \frac{3}{t+1}$$

$t=0,\ v=2$: $\quad 2 + c = 3$

$$\therefore\ c = 1$$

hence $V = \dfrac{3}{t+1} - 1$

b) $v = \frac{3}{t+1} - 1 = 0$

$\frac{3}{t+1} = 1$

$3 = t + 1$

$t = 2$

$v = \frac{dx}{dt} = \frac{3}{t+1} - 1$

$\int (1)\,dx = \int \left(\frac{3}{t+1} - 1\right) dt$

$x = \int \frac{3}{t+1} - 1 \; dt = 3\ln|t+1| - t + c$

$x=0,\ t=0$: $0 = 3\ln 1 - 0 + c$

$\therefore c = 0$

so $x = 3\ln|t+1| - t$

$t=2$: $x = 3\ln 3 - 2 = 1.296\ldots$

$= \boxed{1.3m}$ (2sf)

(Q7a) $T = 2mg$

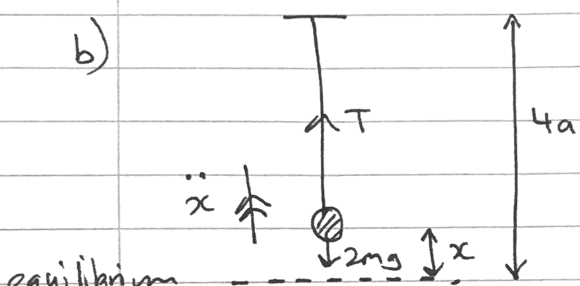
$$T = \frac{\lambda x}{l} = \frac{6mge}{3a} = \frac{2mge}{a}$$

$$\therefore 2mg = \frac{2mge}{a}$$

$$\therefore \frac{e}{a} = 1 \qquad \therefore \boxed{e = a}$$

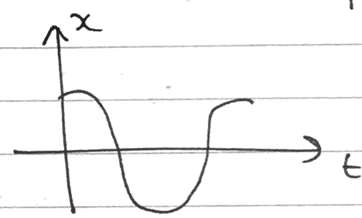
$$\text{so} \quad AO = \boxed{4a}$$

b)



finding where $\ddot{x}$ is positive:

P starts at an endpoint so $x = a\cos\omega t$ applies.



as can be seen, x is max at $t = 0$. this means that upwards is where x is increasing. so $\ddot{x}$ is also increasing upwards hence $\ddot{x}$ is positive upwards

$\overset{+}{\uparrow}$ N2L(P): $T - 2mg = 2m\ddot{x}$

$$T = \frac{6mg}{3a}(4a - x - 3a)$$

$$T = \frac{2mg}{a}(a - x)$$

$$T = 2mg - \frac{2mgx}{a}$$

$$\therefore \cancel{2mg} - \frac{2mgx}{a} - \cancel{2mg} = 2m\ddot{x}$$

$$\Rightarrow -\frac{gx}{a} = \ddot{x} \qquad \text{hence P moves with S.H.M.}$$

//

$\therefore \omega = \sqrt{\frac{g}{a}} \longrightarrow T = 2\pi \times \frac{1}{\omega} = 2\pi\sqrt{\frac{a}{g}}$

c) $a = \frac{a}{4}$: $V_{max} = a\omega = \frac{a}{4} \times \sqrt{\frac{g}{a}} = \boxed{\frac{1}{4}\sqrt{ag}}$ //

d) $x = \frac{a}{4}\cos\omega t$

$x = \frac{a}{4}\cos\left(t\sqrt{\frac{g}{a}}\right)$

$\frac{a}{8} = \frac{a}{4}\cos\left(t\sqrt{\frac{g}{a}}\right)$

$\frac{1}{2} = \cos\left(t\sqrt{\frac{g}{a}}\right)$

$\cos^{-1}\left(\frac{1}{2}\right) = t\sqrt{\frac{g}{a}} = \frac{\pi}{3}$

$t = \frac{\pi}{3}\sqrt{\frac{a}{g}}$ //

$\dot{x} = -\frac{a}{4}\sqrt{\frac{g}{a}}\sin\left(t\sqrt{\frac{g}{a}}\right)$

at $t = \frac{\pi}{3}\sqrt{\frac{a}{g}}$, $\dot{x} = -\frac{a}{4}\sqrt{\frac{g}{a}} \times \frac{\sqrt{3}}{2} = \frac{-\frac{1}{4}\sqrt{ag} \times \sqrt{3}}{2}$

$= -\frac{1}{8}\sqrt{3ag}$

so $|\dot{x}| = \frac{\sqrt{3ag}}{8}$

suvat to highest point :

$$\left.\begin{aligned} s &= s \\ u &= \frac{\sqrt{3ag}}{8} \\ v &= 0 \\ a &= -g \\ t &= \end{aligned}\right\}\quad \begin{aligned} v^2 &= u^2 + 2as \\ 0 &= \frac{3ag}{64} - 2gs \end{aligned}$$

$$s = \frac{3a\cancel{g}}{64 \times 2\cancel{g}} = \frac{3a}{128}$$

$\therefore$ greatest height above O $= \frac{3a}{128} + \frac{a}{8}$

$$= \boxed{\frac{19a}{128}}$$