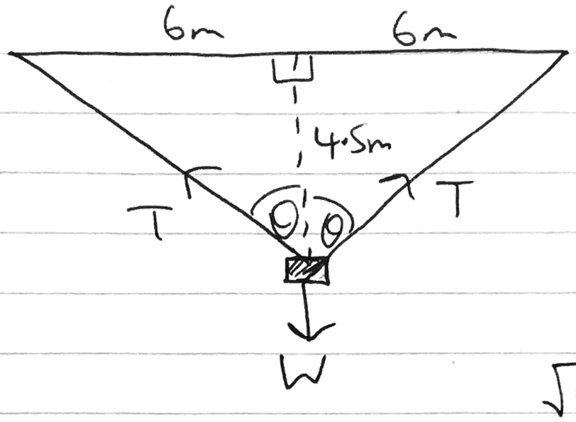
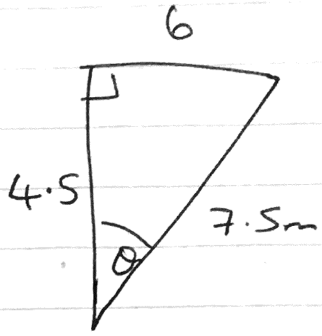
M3 June 2009 (MA)

Q1a)





$\sqrt{4.5^2+6^2} = 7.5$ //

$$\underline{\text{N2L(particle)}} \uparrow^{+} : 2T\cos\theta - W = 0$$

$$2T\cos\theta = W$$

for one half of the string. $\rightarrow$ $T = \dfrac{\lambda x}{l} = \dfrac{80}{4}(7.5-4) = 70N$ //

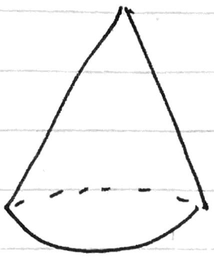
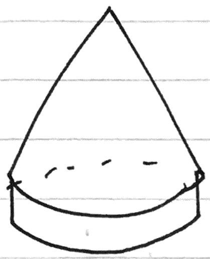
$$\text{and } \cos\theta = \frac{4.5}{7.5} = \frac{9}{15} = \frac{3}{5}$$

$$\therefore \quad W = 2(70)\left(\frac{3}{5}\right) = \boxed{84N}$$

b) $\text{EPE} = \dfrac{\lambda x^2}{2l} = \dfrac{80}{16}(15-8)^2 = \boxed{245N}$

for entire string $\nearrow$

Q2a)

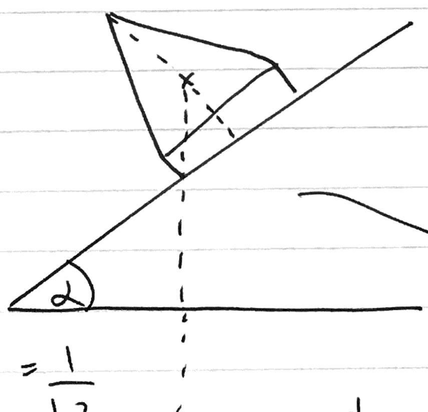
Shape	Mass (vol. ratio)	Distance of c.o.m from surface
	$\boxed{m}$	$2h + \frac{1}{3}\cdot 9h = 3h + 2h = \boxed{5h}$
	$\boxed{3m}$	$\boxed{h}$
	$\boxed{4m}$	$\boxed{\bar{y}}$

taking moments about horizontal surface . . .

$$m(5h) + 3m(h) = 4m(\bar{y})$$

$$\bar{y} = \frac{8mh}{4m} = \boxed{2h}$$

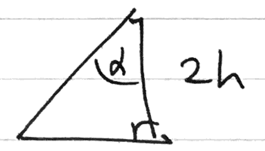
b)



$\tan\alpha = \frac{1}{12}$

'Downward vertical'

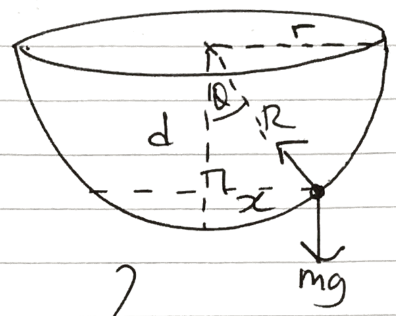
on point of toppling ∴ downward vertical will pass through the point of lowest contact with the plane



$$\tan\alpha = \frac{1}{12} = \frac{r}{2h}$$

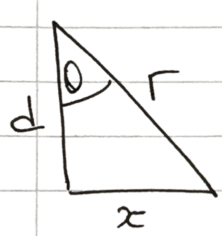
$\frac{r}{2h} = \frac{1}{12}$ $\therefore$ $r = \frac{h}{6}$ $\therefore$ $\boxed{h = 6r}$

Q3a)



$R(\updownarrow): R\cos\theta = mg$ ~ ①

$R(\leftrightarrow): R\sin\theta = mx\omega^2.$ ~ ②
[N2L]



$\therefore \sqrt{x^2 + d^2} = r$

$\therefore r^2 = x^2 + d^2$

$\therefore$

so $\sin\theta = \frac{x}{r}$, $\cos\theta = \frac{d}{r}$

$\Rightarrow R\sin\theta = mx\omega^2$ ~ ②

$\Rightarrow \frac{R\cancel{x}}{\cancel{r}} = m\cancel{x}\left(\frac{3g}{2\cancel{r}}\right)$

$\Rightarrow \boxed{R = \frac{3mg}{2}}$

b) ① : $R = \frac{3mg}{2} = \frac{mg}{\cos\theta} = \frac{mg\,r}{d}$

$\therefore \frac{3}{2} = \frac{r}{d} \longrightarrow \boxed{d = \frac{2r}{3}}$

Q4a) $V = \pi \int_{\frac{1}{4}}^{1} y^2 \, dx = \pi \int_{\frac{1}{4}}^{1} x^{-4} \, dx$

$= \pi \left[-\frac{x^{-3}}{3} \right]_{\frac{1}{4}}^{1} = \pi \left[-\frac{1}{3} \right] - \pi \left[-\frac{64}{3} \right]$

$= \pi \left[\frac{64}{3} - \frac{1}{3} \right] = \boxed{21\pi}$

b) $M\bar{x} = p\pi \int_{\frac{1}{4}}^{1} y^2 x \, dx = p\pi \int_{\frac{1}{4}}^{1} (x^{-3}) \, dx$

$= p\pi \int_{\frac{1}{4}}^{1} \left[\frac{1}{x^3} \right] dx = p\pi \left[-\frac{x^{-2}}{2} \right]_{\frac{1}{4}}^{1}$

$= p\pi \left[-\frac{1}{2} \right] - p\pi \left[-8 \right] = p\pi \left(\frac{15}{2} \right) = M\bar{x}$

$M = 21p\pi$ (from (a)).

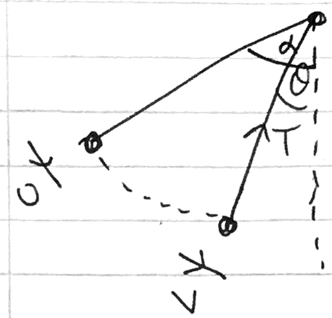
$\therefore \bar{x} = \frac{M\bar{x}}{M} = \frac{\frac{15p\cancel{\pi}}{2}}{21p\cancel{\pi}} = \frac{15}{42} = \boxed{\frac{5}{14}}$

Since the solid is rotated about the x-axis we can deduce $\bar{y} = 0$.

$\therefore \left(\frac{5}{14}, 0 \right)$

$\cos\alpha = \frac{1}{4}$

Q5a)



Energy to find V

$$\underbrace{\frac{mV^2}{2}}_{\Delta KE} = \underbrace{mgl(\cos\theta - \cos\alpha)}_{\Delta GPE}$$

$$\frac{V^2}{2} = gl\left(\cos\theta - \frac{1}{4}\right)$$

$$\therefore \quad V^2 = 2gl\left(\cos\theta - \frac{1}{4}\right)$$

At an angle θ : $\quad$ N2L(p) $\nearrow^{+}$: $T - mg\cos\theta = \frac{mV^2}{l}$

$$\Rightarrow \quad T = mg\cos\theta + \frac{m}{l}(2gl)\left(\cos\theta - \frac{1}{4}\right)$$

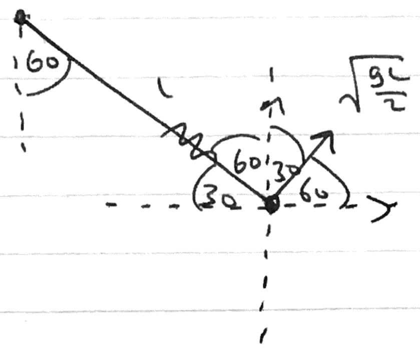
$$\Rightarrow \quad T = mg\cos\theta + 2mg\cos\theta - \frac{2mg}{4}$$

$$\Rightarrow \quad T = 3mg\cos\theta - \frac{mg}{2}$$

b) $\theta = 60°$: $\cos 60 = \frac{1}{2}$.

$$\therefore V^2 = 2gl\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{gl}{2}$$

$$\therefore \quad V = \sqrt{\frac{gl}{2}} .$$



$\therefore u\uparrow = \cos 30\sqrt{\frac{gl}{2}}$.

<u>Suvat to highest point:</u>

$\uparrow +$ $s = x$
$u = \cos 30\sqrt{\frac{gl}{2}}$
$v = 0$
$a = -g$
$t =$

$$v^2 = u^2 + 2as$$

$$0 = \cos^2 30\left(\frac{\cancel{g}l}{2}\right) - 2\cancel{g}x$$

$$x = \frac{l\cos^2 30}{4} = \frac{3l}{16}.$$

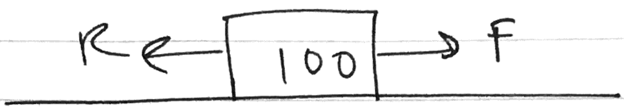
so distance travelled $(\uparrow) = \frac{3l}{16}$.

$\therefore$ distance below vertical $= l\cos 60 - \frac{3l}{16}$

$$= \frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}$$

hence $d = \frac{5l}{16}$

Q6a)



$P = Fv$

$80 = F(v)$

$F = \frac{80}{v}$

$R = kv^2$

$\longrightarrow +$

N2L(system) : $\frac{80}{v} - kv^2 = 100a$

$\Rightarrow \frac{80}{v} - kv^2 = 100v\frac{dv}{dx}$

$\Rightarrow \frac{80 - kv^3}{100v} = v\frac{dv}{dx} = a$

$v = 20,\ a = 0$: $\frac{80 - k(20)^3}{100(20)} = 0$

her max speed is $20ms^{-1}$.

$\therefore 80 = k(20)^3$

$k = \frac{80}{(20)^3} = \frac{1}{100}$ //

so $v\frac{dv}{dx} = \frac{80 - \frac{1}{100}v^3}{100v}$ $\begin{matrix}(\times 100)\\(\times 100)\end{matrix}$

$\Downarrow$

$v\frac{dv}{dx} = \frac{8000 - v^3}{10000v}$ //

b) $\left(\dfrac{10000v^2}{8000-v^3}\right)\dfrac{dv}{dx} = 1$

$$10000\int \frac{v^2}{8000-v^3}\,dv = \int (1)\,dx$$

By Pattern

$$10000\left[-\frac{1}{3}\ln|8000-v^3|\right] = x + c$$

$$-\frac{10000}{3}\ln|8000-v^3| = x + c$$

$\underline{x=0,\ v=4}$: $\quad c = -\dfrac{10000}{3}\ln 7936.$

$$\therefore\ -\frac{10000}{3}\ln|8000-v^3| = x - \frac{10000}{3}\ln 7936$$

$\underline{v=8}$: $\quad x = \dfrac{10000}{3}\ln\left|\dfrac{7936}{8000-(8)^3}\right| = 193.7..m$

$= \boxed{194m}$

c) $a = \frac{8000 - v^3}{10000v} = \frac{dv}{dt}$

$\int_4^8 \frac{10000v}{8000 - v^3}\, dv = [t]_{v=4}^{v=8}$

put v into to obtain values.

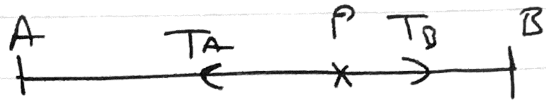
t	$\frac{40000}{7936}$	$\frac{7500}{973}$	$\frac{1250}{117}$
v	4	6	8

$h = \frac{b-a}{n} = \frac{8-4}{2} = 2$

$\therefore$ Area $\simeq \frac{1}{2} \times 2\left[\frac{4000}{7936} + 2\left(\frac{7500}{973}\right) + \frac{1250}{117}\right] \simeq 31.14\ldots$

hence $\boxed{T = 31s}$ to 2s.f.

Q7a)



$T_A = T_B$

$\frac{16}{2}(AP - 2) = \frac{12}{1}(5 - AP - 1)$

$8(AP - 2) = 12(4 - AP)$

$8AP - 16 = 48 - 12AP$

$20AP = 64$

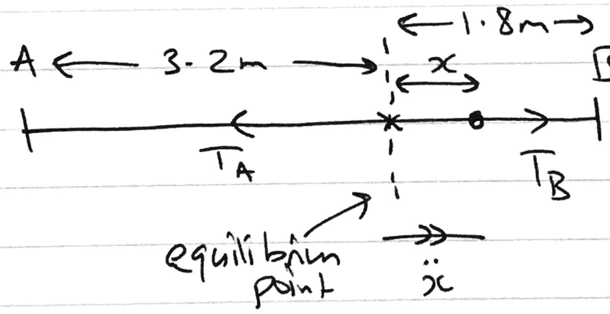
$\therefore AP = 3.2m$

$\hookrightarrow$ extension $= \boxed{1.2m}$

$\therefore BP = 5 - 3.2 = 1.8$

$\hookrightarrow$ extension $= \boxed{0.8m}$

b)



x = asin(ωt) applies as P starts at the centre. So x increases in the direction $\longrightarrow$

$T_B - T_A = 0.5\ddot{x}$

$\frac{12}{1}(1.8 - x - 1) - \frac{16}{2}(3.2 + x - 2) = \frac{1}{2}\ddot{x}$

$12(0.8 - x) - 8(1.2 + x) = 0.5\ddot{x}$

$-8x + 9.6 - 12x - 9.6 = 0.5\ddot{x}$

$-20x = 0.5\ddot{x} \quad \therefore \ddot{x} = -40x$

$\therefore$ P moves with S.H.M

c) $\omega^2 = 40 \quad \therefore \omega = \sqrt{40}$.

so $T = \dfrac{2\pi}{\sqrt{40}}$

$V_{max} = a\omega = \sqrt{40}\, a = \sqrt{10}$

$\therefore a = \dfrac{\sqrt{10}}{\sqrt{40}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$ //

$\Rightarrow \quad x = \frac{1}{2}\sin\left(t\sqrt{40}\right)$

$\Rightarrow \quad \frac{1}{4} = \frac{1}{2}\sin\left(t\sqrt{40}\right)$

$\Rightarrow \quad \frac{1}{2} = \sin\left(t\sqrt{40}\right)$

this is when P returns to being 0.25m from equiibrim.

$\Rightarrow \underbrace{\frac{5\pi}{6}, \frac{\pi}{6}}_{\text{first two solutions}} = t\sqrt{40} \quad \therefore t = \dfrac{\pi}{6\sqrt{40}}, \dfrac{5\pi}{6\sqrt{40}}$ //

So for half of one oscillation, the time that P is not within 0.25m of equiibrim position is $\dfrac{5\pi}{6\sqrt{40}} - \dfrac{\pi}{6\sqrt{40}} = \dfrac{4\pi}{6\sqrt{40}}$ //

So for the entire oscillation, total time outside '0.25m from equilibrium' $= \frac{4\pi}{3\sqrt{40}}$ //

$\therefore$ time <u>within</u> 0.25m $= T - \frac{4\pi}{3\sqrt{40}}$

$= \frac{2\pi}{\sqrt{40}} - \frac{4\pi}{3\sqrt{40}}$

$= \frac{6\pi - 4\pi}{3\sqrt{40}}$

$= \frac{2\pi}{3\sqrt{40}}$

$\therefore$ proportion $= \dfrac{\frac{2\pi}{3\sqrt{40}}}{T} = \dfrac{\frac{2\pi}{3\sqrt{40}}}{\frac{2\pi}{\sqrt{40}}}$

$= \boxed{\frac{1}{3}}$