M3 - June 2007

1- a) $A = \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{1}{3}x^3\right]_0^2 = 4 - \frac{8}{3}$

$= \frac{4}{3}$ units^2

b) By symmetry $\bar{x} = 1$

$\frac{4}{3}\bar{y} = \int_0^2 \frac{1}{2}y^2\,dx = \frac{1}{2}\int_0^2 (2x - x^2)^2\,dx = \int_0^2 \frac{1}{2}(4x^2 - 4x^3 + x^4)\,dx$

$= \frac{1}{2}\left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right]_0^2 = \frac{1}{2}\left(\frac{32}{3} - 16 + \frac{32}{5}\right) = \frac{16}{15} \times \frac{1}{2} = \frac{8}{15}$

$\bar{y} = \frac{3 \times 8}{4 \times 15} = \frac{2}{5}$

$\therefore$ Centre of mass of lamina is at $\left(1, \frac{2}{5}\right)$

2- a) Com of cylinder: $\frac{1}{2}h$ from O

$\frac{1}{2}h \times 2\pi h^2 h = (2\pi h^2 h + \pi h^2)\bar{d}$

$\frac{1}{2}h \times 2 = (2+1)\bar{d}$

$h = 3\bar{d}$

$\bar{d} = \frac{1}{3}h$

b) $\frac{1}{3}hM + \frac{1}{2}hM = 2M\bar{d}$

$\frac{5}{6}h = 2\bar{d}$

$\bar{d} = \frac{5}{12}h$

3- a) $F = \frac{k}{x^2}$ When $x = R$, $F = gm$ $gm = \frac{k}{R^2}$ $k = mgR^2$

b) $\uparrow [F = ma]$

$-\frac{mgR^2}{x^2} = mv\frac{dv}{dx}$

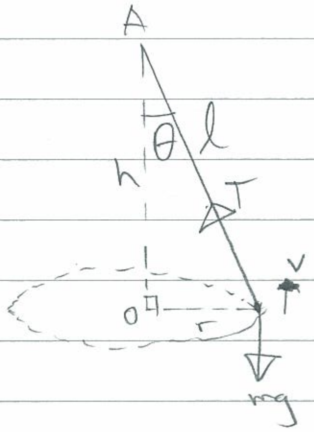
$-gR^2 \int_{2R}^{R} x^{-2}\,dx = \int_0^v v\,dv$

$+gR^2\left[\frac{1}{x}\right]_{2R}^{R} = \frac{1}{2}\left[v^2\right]_0^v$

$gR - \frac{1}{2}gR = \frac{1}{2}v^2$

$v^2 = gR$ $v = \sqrt{gR}$ ms^{-1}

4.



$\uparrow \; mg = T\cos\theta \qquad \leftarrow [F = ma]$

$T = \dfrac{mg}{\cos\theta}$ ① $\qquad T\sin\theta = \dfrac{mv^2}{r}$ ②

① in ②: $\dfrac{mg}{\cos\theta}\cdot\sin\theta = \dfrac{\cancel{m}v^2}{r}$

$g\tan\theta = \dfrac{v^2}{r}$ ③

$\tan\theta = \dfrac{r}{h} = \dfrac{r}{\sqrt{l^2 - r^2}}$ ③ $\qquad$ ③ in ③: $\dfrac{gr}{\sqrt{l^2-r^2}} = \dfrac{v^2}{r}$

$gr^2 = v^2\sqrt{l^2 - r^2}$

5. a) $\ddot{x} = -\omega^2 x$

$1 = |-\omega^2 \cdot 0.04|$

$\omega^2 = 25$

$\omega = 5$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5}$

b) $v^2 = \omega^2(a^2 - x^2)$

$0.2^2 = 25(a^2 - 0.04^2)$

$0.04 = 25a^2 - 0.04$

$a^2 = \dfrac{0.08}{25}$

$a = \dfrac{\sqrt{2}}{25}$ m

c) $x = a\sin 5t$

$\pm\frac{1}{2}a = a\sin 5t$

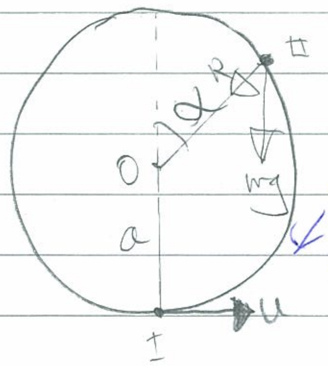
$\pm\frac{1}{2} = \sin 5t$

$5t = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$t = \dfrac{\pi}{30}, \dfrac{5\pi}{30}, \dfrac{7\pi}{30}, \dfrac{11\pi}{30}$

$T_{+ve} = \left(\dfrac{5\pi}{30} - \dfrac{\pi}{30}\right) + \left(\dfrac{11\pi}{30} - \dfrac{7\pi}{30}\right) = \dfrac{4\pi}{15}$

6.



a) $ME_I = ME_{II}$

$\frac{1}{2}\cancel{m}u^2 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(a + a\cos\alpha)$

$u^2 = v^2 + 2ga + 2ga\cos\alpha$ ①

$\swarrow [F = ma] \quad R + mg\cos\alpha = \dfrac{mv^2}{r}$

$mg\cos\alpha = \dfrac{mv^2}{a}$

$v^2 = ag\cos\alpha$ ②

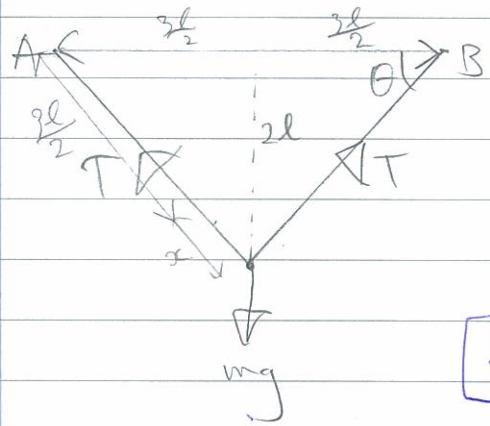
③ in ②: $U^2 = ag\cos\alpha + 2ga + 2ga\cos\alpha$
$= 3ag\cos\alpha + 2ag = ag(2 + 3\cos\alpha)$

b) $ME_f = ME_0$

$\frac{1}{2}mU^2 = \frac{1}{2}mW^2 + mga$

$W^2 = U^2 - 2ga = 2ga + 3ga\cos\alpha - 2ga$
$= 3ga \times \frac{1}{\sqrt{3}} = \frac{3ag}{\sqrt{3}} = ag\sqrt{3}$

7.



a) ↑ $mg = 2T\sin\theta$ ①

$\frac{3l}{2} + x = \sqrt{(2l)^2 + \left(\frac{3l}{2}\right)^2} = \sqrt{4l^2 + \frac{9}{4}l^2}$

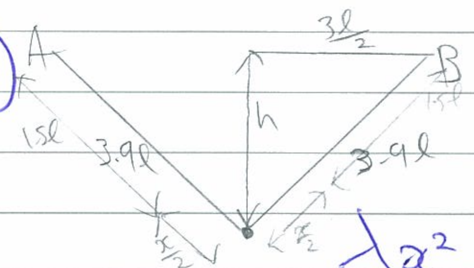
$x = \frac{5}{2}l - \frac{3l}{2} = l$

$\boxed{HL}$ $T = \frac{\lambda x}{a} = \frac{\lambda l \cdot 2}{3l} = \frac{2\lambda}{3}$ ②

② in ①: $mg = 2 \times \frac{2\lambda}{3}\sin\theta$ $\sin\theta = \frac{2l}{\frac{5l}{2}} = \frac{4}{5}$ ④

$\frac{3mg}{4\sin\theta} = \lambda$ ③   ④ in ③: $\lambda = \frac{3mg}{4 \times \frac{4}{5}} = \frac{15mg}{16}$

b)



$h^2 = (3.9^2 - 1.5^2)l^2 = 12.96l^2$
$h = 3.6l$

$\frac{\lambda x^2}{2a} = mgh + \frac{1}{2}mv^2$

$x = (7.8 - 3)l$
$= 4.8l$

$\frac{15mg}{16} \cdot \frac{(4.8l)^2}{6l} = mg \times 3.6l + \frac{1}{2}mv^2$

$v^2 = \frac{15 \times 4.8^2 \times 2lg}{16 \times 6} - 2 \times 3.6lg = 70.56l - 70.56l$
$= 0$