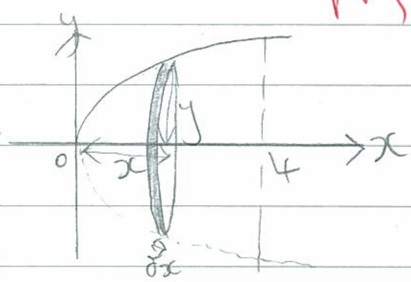
M3 - June 2006

1-



$$\pi\int_0^4 y^2 x\,dx = \pi\int_0^4 y^2\,dx \times \bar{d}$$

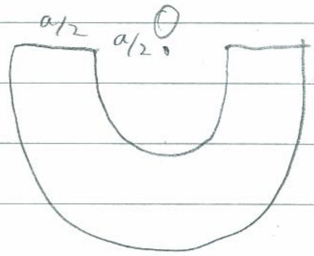
$$\int_0^4 x^2\,dx = \int_0^4 x\,dx \times \bar{d}$$

$$\frac{1}{3}\left[x^3\right]_0^4 = \frac{1}{2}\left[x^2\right]_0^4 \bar{d}$$

$$\frac{64}{3} = 8\bar{d}$$

$$\bar{d} = \frac{8}{3}$$

2-



a) $\frac{3}{8}a\cdot\frac{2}{3}\pi a^3 - \frac{3}{8}\cdot\frac{a}{2}\cdot\frac{2}{3}\pi\frac{a^3}{8} = \left(\frac{2}{3}\pi a^3 - \frac{2}{3}\pi\frac{a^3}{8}\right)$

$$\frac{1}{4}a - \frac{1}{64}a = \frac{7}{12}\bar{d}$$

$$\bar{d} = \frac{45}{112}a$$

b) $\frac{45}{112}\cancel{a}m + \frac{3}{8}\cdot\frac{a}{2}\cdot km = (m+km)\frac{17}{48}\cancel{a}$

$$\frac{45}{112}m + \frac{3}{16}km = \frac{17}{48}m + \frac{17}{48}km$$

$$\frac{1}{21}m - \frac{1}{6}km = 0$$

$$m\left(\frac{1}{21} - \frac{1}{6}k\right) = 0$$

$$k = \frac{6}{21} = \frac{2}{7}$$

3- a) $f = 5$, $r = 0.1$, $w = 2\pi f = 10\pi$

$$a_{max} = w^2 r = (10\pi)^2 \times 0.1 = 10\pi^2$$

$[F = ma]$

$$F_{max} = 0.2\times 10\pi^2 = 19.7\text{ N (3sf)}$$

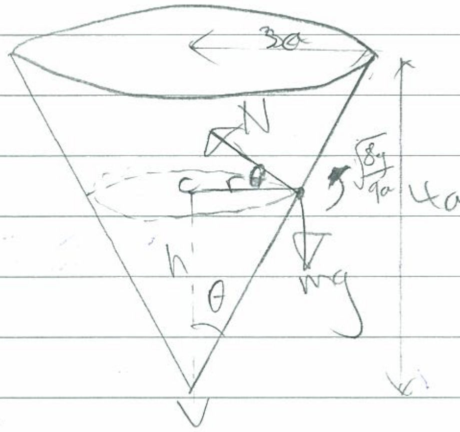
b) $f = 5, \; r = 0.2 \quad w = 2\pi f = 10\pi$

~~$v_{max} = wr = 10\pi \times 0.2 = 2\pi = 6.28\,ms^{-1}$ (3sf)~~

$v^2 = w^2(a^2 - x^2) = 10^2\pi^2(0.2^2 - 0.1^2)$

$v = 5.44\,ms^{-1}$ (3sf)

4.



$\boxed{F = ma}$

$N\cos\theta = m w^2 r$

$N\cos\theta = m\frac{8g}{9a} \cdot r$

$N \cdot \frac{4}{5} = m\frac{8g}{9a} \cdot r$

$r = \frac{9aN}{10mg}$ ①

$\uparrow \; mg = N\sin\theta = N \cdot \frac{3}{5}$

$N = \frac{5mg}{3}$ ② $\quad$ ② in ①: $\; r = \frac{9a \cdot 5mg}{3 \times 10mg} = \frac{3}{2}a$

$\tan\theta = \frac{r}{h} = \frac{3}{4}$

$\frac{3a}{2h} = \frac{3}{4}$

$h = 2a \quad \therefore$ Height of C above V is $2a$

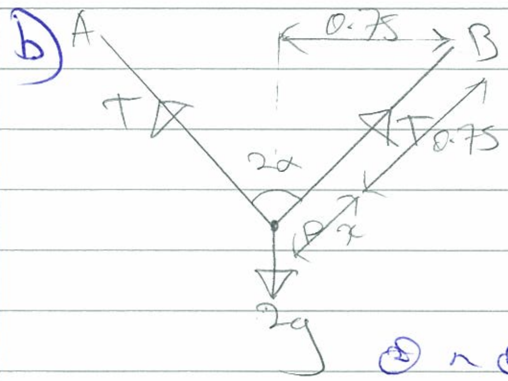
5. a)

$0.75 + x = \sqrt{1^2 + 0.75^2} = 1.25$

$x = 0.5$

$\cancel{2} \times g \times 1 = \frac{1}{\cancel{2}} \times \cancel{2} \times v^2 + \frac{49 \times 0.5^2}{\cancel{2} \times 0.75} \times \cancel{2}$

$v^2 = 2g - 16\frac{1}{3} \quad v = 1.81\,ms^{-1}$ (3sf)

b) 

$\uparrow \; 2T\cos\alpha = 2g$

$T\cos\alpha = g$ ①

$T = \dfrac{\lambda x}{a} = \dfrac{49x}{0.75} = 65\tfrac{1}{3}x$ ②

② in ①: $196x\cos\alpha = 3g$ $x = \dfrac{3}{20\cos\alpha}$ ③

$20x\cos\alpha = 3$

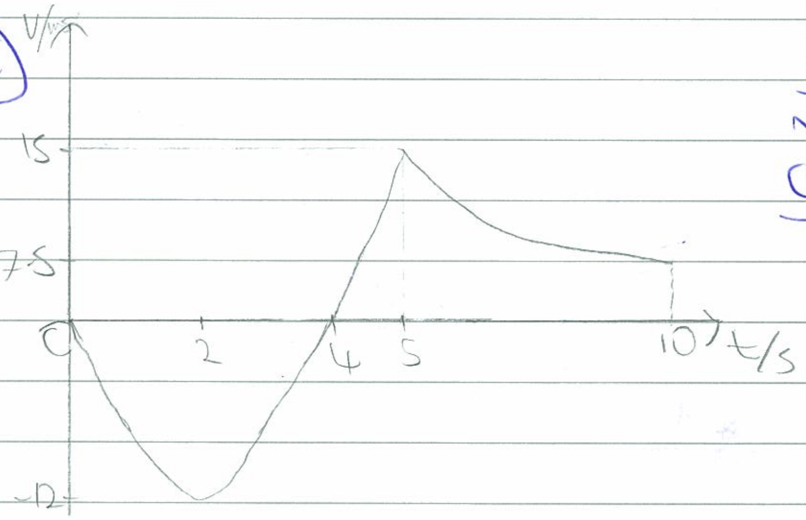
$\sin\alpha = \dfrac{0.75}{0.75 + x}$ ④

③ in ④: $\sin\alpha = \dfrac{0.75}{0.75 + \frac{3}{20\cos\alpha}}$

$\frac{3}{4}\sin\alpha + \dfrac{3\sin\alpha}{20\cos\alpha} = \dfrac{3}{4}$

~~$15\sin\alpha + 3\tan\alpha = 15\cos\alpha$~~

$5\sin\alpha + \tan\alpha = 5$

6. a) 

b) $2 < t < 5$

c) Distance $= \int_0^4 3t^2 - 12t \, dt + \int_4^5 3t^2 - 12t$

$= -3\left[\frac{1}{3}t^3 - 2t^2\right]_0^4 + 3\left[\frac{1}{3}t^3 - 2t^2\right]$

$= -3\left(\frac{64}{3} - 32\right) + 3\left(\frac{125}{3} - 50 - \frac{64}{3} + \right.$

$= 32 + 7 = 39$

d) Distance in 4 seconds = 32 m

$32 - 7 = 25$ m

$25 = \int_5^t 75t^{-1} \, dt$

$\frac{1}{3} = [\ln t]_5^t$

$\frac{1}{3} = \ln t - \ln 5$

$\ln t = \frac{1}{3} + \ln 5$

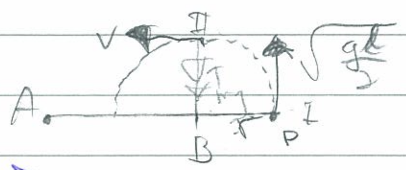
$t = 6.98$ s (3 s.f.)

7. a) $\frac{1}{2} \cdot m \cdot \frac{5gl}{2} = m \cdot g \cdot l \cdot 4 + \frac{1}{2} mv^2 \cdot 2$

$$5gl = 4gl + 2v^2$$

$$v^2 = \frac{gl}{2}$$

$$v = \sqrt{\frac{gl}{2}}$$

b) 

$ME_{II} = ME_{I}$

$\frac{1}{2} \cdot m \cdot \frac{gl}{2} = mgr + \frac{1}{2} mv^2$

$\frac{gl}{4} = gr + \frac{v^2}{2}$

$v^2 = \frac{gl}{2} - 2gr$

$\downarrow \boxed{F = ma}$

$T + mg = \frac{m}{r}\left(\frac{gl}{2} - 2gr\right)$

$T + mg = \frac{mgl}{2r} - 2mg$

$T = \frac{mgl}{2r} - 3mg$

For complete semicircle, $T \geqslant 0$

$$\frac{mgl}{2r} - 3mg \geqslant 0$$

$$\frac{l}{2r} \geqslant 3$$

$$l \geqslant 6r \quad ①$$

$AB = l - r$

$r = l - AB \quad ②$

$AB \geqslant 2r$

② → ①:

$l \geqslant 6l - 6AB$

$6AB \geqslant 5l$

$AB \geqslant \frac{5}{6} l$