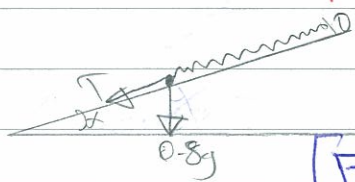


M3 - June 2005

1-



$$T = \frac{\Delta x}{a} = \frac{20 \times 0.4}{2} = 4 \text{ N}$$

$$[F = ma]$$

$$4 + 0.8g \sin \alpha = 0.8a$$

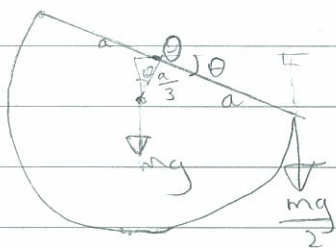
$$a = \frac{4 + 0.8 \times 9.8 \times \frac{3}{5}}{0.8} = 10.88 \text{ ms}^{-2}$$

2 a) $\frac{1}{2} r \cdot 2\pi r^2 = (2\pi r^2 + \pi r^2) d$

$$a = 3d$$

$$d = \frac{1}{3} a$$

b)



$$\frac{1}{2} Mg a \cos \theta = \frac{Mg a \sin \theta}{3}$$

$$3 \cdot \frac{1}{2} = \tan \theta$$

$$\theta = 56^\circ \text{ (2sf)}$$

3



~~$$T_1 \cdot l = 4mg \cdot \frac{4l}{3} + 2mg \cdot \frac{2l}{3} = 16mg \cdot \frac{l}{3} + 4mg \cdot \frac{l}{3} = 20mg \cdot \frac{l}{3}$$~~

$$T_1 = \frac{4mg(l+x)}{l}$$

~~$$T_2 \cdot l = 4mg \cdot \frac{2l}{3} + 2mg \cdot \frac{l}{3} = 8mg \cdot \frac{l}{3} + 2mg \cdot \frac{l}{3} = 10mg \cdot \frac{l}{3}$$~~

$$T_2 = \frac{4mg(l-x)}{l}$$

$$\frac{4mg(l-x)}{l} - \frac{4mg(l+x)}{l} = \ddot{x}$$

$$4g - \frac{4gx}{l} - 4g - \frac{4gx}{l} = \ddot{x}$$

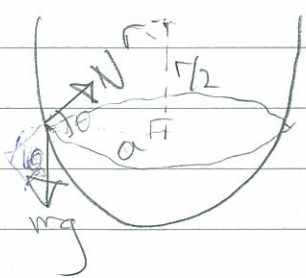
$$-\frac{8gx}{l} = \ddot{x}$$

$$\therefore \text{SHM with } \omega^2 = \frac{8g}{l}$$

$$v = \omega r = \sqrt{\frac{8g}{l}} \times \frac{l}{3} = \frac{2\sqrt{2g}}{3\sqrt{l}} = \frac{2\sqrt{2gl}}{3}$$

c) In SHM, max vel occurs at the centre of oscillation.

4.



a) ~~$N = mg \sin \theta = mg \frac{r}{2}$~~

~~$mg = N \sin \theta$~~

$N = \frac{mg}{\sin \theta} = \frac{mg}{\frac{r}{2} \times \frac{1}{r}} = 2mg$

b) $[F = ma] \rightarrow$

$N \cos \theta = m \omega^2 r$

$N \cos \theta = m \omega^2 r \cos \theta$

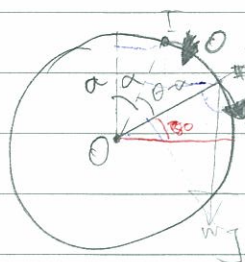
$2mg = m \omega^2 r$

$\omega^2 = \frac{2g}{r}$

$T = \frac{2\pi r}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{r}}}$

$= \frac{2\pi}{\sqrt{\frac{2g}{r}}} \times \sqrt{r} = \frac{\pi \sqrt{2gr}}{g}$

5.



a) $m v_B^2 = m v_A^2$
 $mg(a \cos \alpha - a \cos \theta) = \frac{1}{2} m v^2$

$v^2 = 2ga(\cos \alpha - \cos \theta)$

b) $[F = ma]$

$mg \cos \theta - R = \frac{m}{a} (2ga \cos \alpha - 2ga \cos \theta)$

$mg \cos \theta = 2g \times \frac{3}{4} - 2g \times \cos \theta$

$3g \cos \theta = \frac{3}{2}g$

$\cos \theta = \frac{1}{2}$

$\theta = 60^\circ$

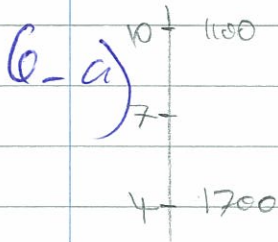
c) $v^2 = 2ga \cdot \frac{3}{4} - 2ga \cdot \frac{1}{2} = \frac{3}{2}ga - ga = \frac{1}{2}ga$

$\frac{1}{2} m v^2 = \frac{1}{2} m v^2 + mg \cdot (a + \frac{a}{2}) = \frac{1}{2} m v^2$

$\frac{1}{2} ga + \frac{3}{2} ga = \frac{1}{2} v^2$

$v^2 = 7ga$

$v = \sqrt{7ga}$



6-a)

$$a = 3\text{m}$$

$$T = 12\text{h}$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{6} \text{ rad h}^{-1}$$

$$x = a \cos \omega t$$

$$x = 3 \cos \frac{\pi}{6} \cdot 5 = 3 \cos \frac{5\pi}{6} = -\frac{3\sqrt{3}}{2}$$

$$v^2 = \omega^2 (a^2 - x^2)$$

$$v^2 = \frac{\pi^2}{36} \left(9 - \frac{27}{4} \right) = \frac{\pi^2}{16}$$

$$v = \frac{\pi}{4} \text{ m h}^{-1}$$

b) $-1.5 = 3 \cos \frac{\pi}{6} t$

$$-\frac{1}{2} = \cos \frac{\pi}{6} t$$

$$\frac{4\pi}{3} / \frac{2\pi}{3} = \frac{\pi}{6} t$$

$t = 4, 8$ \therefore Total time is 4h

7-a) $[F = ma]$

$$-\frac{k}{(x+1)^2} = \frac{1}{3} v \frac{dv}{dx}$$

$$-k \int_1^8 (x+1)^{-2} dx = \frac{1}{3} \int_4^{\sqrt{2}} v dv$$

$$+k \left[(x+1)^{-1} \right]_1^8 = \frac{1}{3} \cdot \frac{1}{2} \left[v^2 \right]_4^{\sqrt{2}}$$

$$k \left(\frac{1}{9} - \frac{1}{2} \right) = \frac{1}{6} (2 - 16)$$

$$+\frac{7}{18} k = +\frac{7}{3}$$

$$k = \frac{18 \cdot 7}{3 \cdot 7} = 6$$

b) $-6 \int_1^x (x+1)^{-2} dx = \frac{1}{3} \int_4^0 v dv$

$$6 \left[(x+1)^{-1} \right]_1^x = \frac{1}{6} \left[v^2 \right]_4^0$$

$$\frac{6}{x+1} - 3 = -\frac{8}{3}$$

$$6 = \frac{x+1}{3}$$

$$x+1 = 18$$

$$x = 17\text{m}$$

