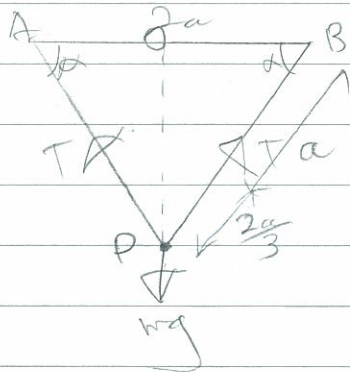


M3 - June 2004

1.  $\omega = 1000 \text{ rev min}^{-1} = \frac{2\pi \times 1000}{60} = 104.7 \text{ rad s}^{-1}$

$v = \omega r = 104.7 \times 3.5 \times 10^{-2} = 3.67 \text{ ms}^{-1} \text{ (3sf)}$

2.



$\cos \alpha = \frac{a}{PB} = \frac{3}{5}$

$PB = \frac{5}{3}a$

$T = \frac{\lambda x}{a}$

$= \frac{\lambda \cdot 2a}{3a}$

$\tan \alpha = \frac{OP}{a} = \frac{4}{3}$

$= \frac{2\lambda}{3}$

$OP = \frac{4a}{3}$

$mg \cdot \frac{4a}{3} = \lambda \left(\frac{2a}{3}\right)^2 \cdot 2$

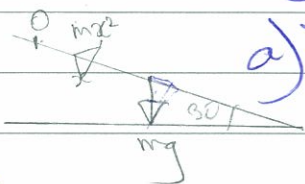
$\frac{4mga}{3} = \frac{8a^2 \lambda}{18a}$

$\frac{4mg}{3} = \frac{8\lambda}{18}$

$mg = \frac{\lambda}{3}$

$\lambda = 3mg$

3.



a)  $\boxed{F = ma}$

$mg \sin 30 - mx^3 = m \frac{v dv}{dx}$

b)  $\int_0^x (4.9 - x^3) dx = \int_0^v v dv$

$\left[4.9x - \frac{1}{3}x^3\right]_0^x = \left[\frac{1}{2}v^2\right]_0^v$

$4.9x - \frac{2}{3}x^3 = 0$

$x(4.9 - \frac{2}{3}x^2) = 0$

$x = 0$  or  $4.9 - \frac{2}{3}x^2 = 0$

$x^2 = 14.7$

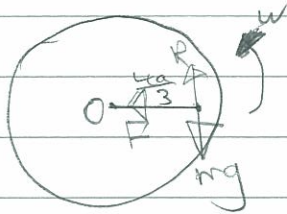
$x = 3.8 \text{ m}$

$9.8 - \frac{8}{3} = \frac{1}{2}v^2$

$v^2 = \frac{214}{5}$

$v = 3.78 \text{ ms}^{-1}$

4.



a)  $\uparrow mg = R$

$\leftarrow [F = ma]$   
 $F = \frac{m\omega^2 \cdot 4a}{3}$

$F \leq \mu R$

$\frac{m\omega^2 \cdot 4a}{3} \leq \frac{3}{5} mg$

$4a\omega^2 \leq \frac{9g}{5}$

$\omega^2 \leq \frac{9g}{20a}$

b)  $T = \frac{I\alpha}{a} = \frac{2mga}{3a}$   
 $= \frac{2mg}{3}$

$\leftarrow [F = ma]$   
 $F + \frac{2mg}{3} = \frac{m\omega^2 \cdot 4a}{3}$

$F = \frac{m\omega^2 \cdot 4a}{3} - \frac{2mg}{3}$

$F \leq \mu R$

$\frac{4m\omega^2 \cdot 4a}{3} - \frac{2mg}{3} \leq \frac{3}{5} mg$

$4a\omega^2 - 2g \leq \frac{9g}{5}$

$4a\omega^2 \leq \frac{19g}{5}$   
 $\omega^2 \leq \frac{19g}{20a}$

$\leftarrow [F = ma]$

$\frac{2mg}{3} - F = \frac{m\omega^2 \cdot 4a}{3}$

$F = \frac{2mg - m\omega^2 \cdot 4a}{3}$

$F \leq \mu R$

$\frac{2mg - m\omega^2 \cdot 4a}{3} \leq \frac{3}{5} mg$

$2mg - m\omega^2 \cdot 4a \leq \frac{9mg}{5}$

$\frac{mg}{5} \leq m\omega^2 \cdot 4a$

$\omega^2 \geq \frac{g}{20a}$

$\therefore \frac{g}{20a} \leq \omega^2 \leq \frac{19g}{20a}$

5. a)  $\pi(3r)^2 \cdot 4r \cdot 2r - \frac{1}{3}\pi(3r)^2 \cdot 4r \cdot r = (\pi(3r)^2 \cdot 4r) + \frac{1}{3}\pi(3r)^2 \cdot 4r$

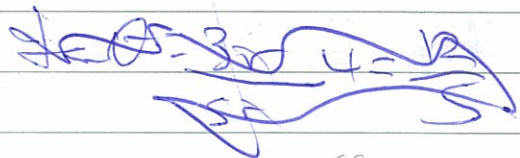
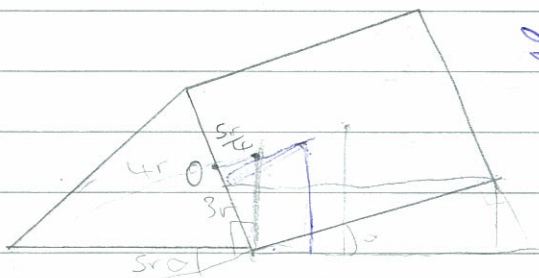
$$8r - \frac{4}{3}r = \left(4 + \frac{4}{3}\right)d$$

$$d = \frac{5r}{4}$$

b)  $\tan \theta = 3r \div \left(4r - \frac{5r}{4}\right) = \frac{3r}{11r} \cdot 4 = \frac{12}{11}$

$\theta = 42.5^\circ$  (1dp)

c)



$$x = \sqrt{\left(\frac{5}{4}r\right)^2 - (3r)^2}$$

$$= \sqrt{\frac{25}{16}r^2 - 9r^2} = 4.3r < 5r$$

$\therefore$  It will not topple

6. a)  $m v_B = m v_C$

$$\frac{1}{2} m 20^2 + m g (50 - 25) = \frac{1}{2} m v^2$$

$$200 + 25g = \frac{1}{2} v^2$$

$$v^2 = 800$$

$$v = 29.8 \text{ m/s (3sf)}$$

b)  $[F = ma]$

$$N - 70g = \frac{70 v^2}{50}$$

$$N = \frac{80000}{50} + 9800 = 19320 \text{ N}$$

$$c) \frac{1}{2} \cancel{mv^2} = 70 + 890 = \frac{1}{2} \cancel{70} v^2 + 70g \times 50 - \left(\frac{\sqrt{3}}{2} 50\right)$$

$$445 = \frac{1}{2} v^2 + 656$$

$$v = 27.5 \text{ ms}^{-1} \text{ (3sf)}$$

$$d) R_c = \frac{70 \times 9.8}{2} = 343 \text{ N}$$

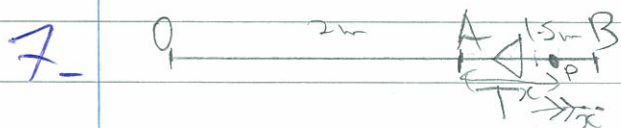
$$\uparrow [F = ma]$$

$$\cancel{R_2} R_2 - 343 = m \frac{20^2}{50}$$

$$R_2 = \frac{70 \times 20 \times 20}{50} + 343 = 903$$

$$\text{change} = 560 \text{ N}$$

e) Lower reaction because of lower speed



$$d) T = \frac{I \alpha}{a} = \frac{2k \cdot 6x}{2} = 10.8x$$

$$[F = ma]$$

$$-T = 0.3 \ddot{x}$$

$$-10.8x = 0.3 \ddot{x}$$

$$\ddot{x} = -36x$$

$$\therefore \text{SHM with } \omega^2 = 36$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ s}$$

$$b) v_{\text{max}} = \omega r = 6 \times 1.5 = 9 \text{ ms}^{-1}$$

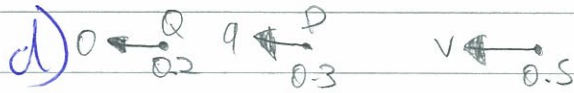
$$c) x = 1.5 \cos 6t$$

$$0.75 = 1.5 \cos 6t$$

$$0.5 = \cos 6t$$

$$6t = \frac{\pi}{3}$$

$$t = \frac{\pi}{18} \text{ s}$$



$$9 \times 0.3 = 0.5v$$

$$v = 5.4 \text{ m/s}^2$$

$$T = \frac{\lambda x}{a} = \frac{21.6x}{2} = 10.8x$$

$$[F = ma]$$

$$-10.8x = 0.5\ddot{x}$$

$$\ddot{x} = -21.6x$$

$$\therefore SHM \text{ with } \omega^2 = 21.6$$

$$v_{\text{max}} = \omega r$$

$$r = \frac{5.4}{\sqrt{21.6}} = 1.16 \text{ m (3sf)}$$

