M3 - June 2003

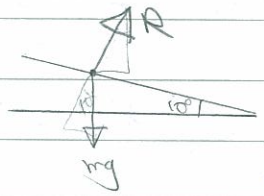
1- $ME_A = MB_B + WD$

$$\frac{\lambda x^2}{2a} = 0 + \frac{2}{3}mg\,AB$$

$$\frac{4mga^2}{2a \times 4} = \frac{2}{3}mg\,AB$$

$$AB = \frac{3a}{4}$$

2-



$R\cos\theta = mg$

$R = \frac{mg}{\cos\theta}$

$[F = ma]$

$R\sin\theta = \frac{m \cdot 18^2}{r}$

$mg\tan\theta = \frac{m324}{r}$

$r = \frac{324}{g\tan\theta} = 190\text{m}$ (2 s.f.)

3- a) $[F = ma]$

$$\int_{10}^{x} x(4-3x)\,dx = 0.2\int_{0}^{v} v\,dv$$

$$\int_{6}^{x} 4x - 3x^2\,dx = 2\int_{0}^{v} v\,dv$$

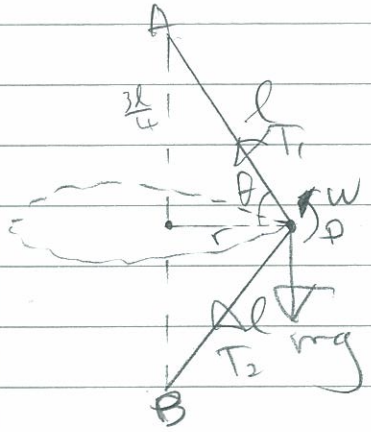
$$\left[2x^2 - x^3\right]_6^x = \left[v^2\right]_0^v$$

$$v^2 = 2x^2 - x^3 + 144$$

b) When $x = 0$, $v^2 = 144$

$v = 12\,ms^{-1}$

4-



a) $\uparrow mg - T_1\sin\theta + T_2\sin\theta = 0$

$T_2\frac{3l}{4l} = T_1\cdot\frac{3l}{4l} - mg$

$T_2 = T_1 - \frac{4mg}{3}$ ①

$\leftarrow$ [F = ma]

$T_1\cos\theta + T_2\cos\theta = mr\omega^2$

$T_1\cos\theta = ml\cos\theta\,\omega^2 - T_2\cos\theta$

$T_1 = ml\omega^2 - T_1 + \frac{4mg}{3}$

$2T_1 = \frac{1}{3}m(3l\omega^2 + 4g)$

$T_1 = \frac{1}{6}m(3l\omega^2 + 4g)$

b) $T_2 = T_1 - \frac{4mg}{3}$

$= \frac{1}{2}ml\omega^2 + \frac{2mg}{3} - \frac{4mg}{3}$

$= \frac{1}{2}ml\omega^2 - \frac{2}{3}mg$

$= \frac{1}{6}m(3l\omega^2 - 4g)$

c) $T_2 \geq 0$

$\frac{1}{6}m(3l\omega^2 - 4g) \geq 0$

$3l\omega^2 \geq 4g$

$\omega^2 \geq \frac{4g}{3l}$

5- a) 

$T = \frac{\lambda x}{d} = \frac{12x}{0.6} = 20x$

[F = ma]

$-T = 0.8\ddot{x}$

$-20x = 0.8\ddot{x}$

$\ddot{x} = -25x$

$\omega^2 = 25$

$\omega = 5$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$ s

b) $a_{max} = \omega^2 r = 25 \times 0.25 = 6.25$ m s^{-2}

c) $x = 0.25\cos 5t$

$v = -1.25\sin 5t$

At $t = 2$, $v = -1.25\sin 10 = 0.68$ m s^{-1}

d) Away from O

6- a) $KE_A = KE_C$

$$\frac{1}{2}mu^2 + mg(a - a\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2ga - 2ga\cos\theta$$

$[F = ma]$

$$mg\cos\theta - R = \frac{mv^2}{r}$$

$$mg\cos\theta = \frac{m}{a}\left(u^2 + 2ga - 2ga\cos\theta\right)$$

$$g\cos\theta = \frac{u^2}{a} + 2g - 2g\cos\theta$$

$$3g\cos\theta = \frac{u^2}{a} + 2g$$

$$\cos\theta = \frac{u^2}{3ga} + \frac{2}{3}$$

b) $KE_A = KE_B$

$$\frac{1}{2}mu^2 + m2ag = \frac{1}{2}m\frac{9ag}{2}$$

$$u^2 = \frac{9ag}{2} - 4ag = \frac{ag}{2}$$

$$\cos\theta = \frac{ag}{2\times 3ga} + \frac{2}{3} = \frac{1}{6} + \frac{3}{3} = \frac{5}{6}$$

$$\theta = 34^\circ \ (2sf)$$

7- a) $V = \pi\int_0^2 y^2\,dx = \frac{\pi}{4}\int_0^2 (x-2)^4\,dx = \frac{\pi}{4}\left[\frac{1}{5}(x-2)^5\right]_0^2$

$$= \frac{\pi}{20}(0+32) = \frac{8\pi}{5}\ cm^3$$

b) $\pi\int_0^2 y^2 x\,dx = \frac{8\pi}{5}\bar{x}$

$\frac{\pi}{4}\int_0^2 x(x-2)^4\,dx = \frac{8\pi}{5}\bar{x}$

Let $u = x$ $\quad \frac{dv}{dx} = (x-2)^4$

$\frac{du}{dx} = 1$ $\quad v = \frac{1}{5}(x-2)^5$

$\frac{32\bar{x}}{5} = \left[\frac{x}{5}(x-2)^5\right]_0^2 - \frac{1}{5}\int_0^2 (x-2)^5\,dx$

$= -\frac{1}{5\times 6}\left[(x-2)^6\right]_0^2$

$= -\frac{1}{30}(-64) = \frac{32}{15}$

$\frac{32\bar{x}}{5} = \frac{32}{15}$

$\bar{x} = \frac{5\times 32}{15\times 32} = \frac{1}{3}$ cm

c) B) $\frac{1}{3}\times 2W + 8F - 4\times 10W = 0$

$\frac{2}{3}W + 8F - 40W = 0$

$8F = \frac{118}{3}W$

$F = \frac{59}{12}W$