M3 - June 2002

1 - $T = \frac{2\pi}{w}$

$w = \frac{2\pi}{2} = \pi$

$x = a\cos wt$

$x = 0.25\cos(\pi t)$

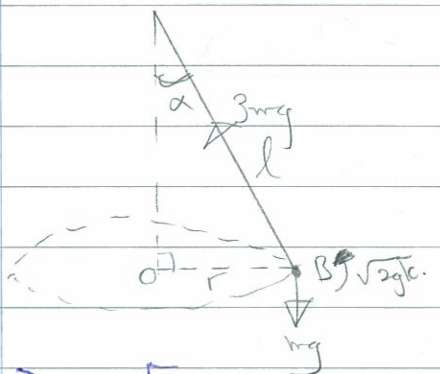
$-0.125 = 0.25\cos(\pi t)$

$-0.5 = \cos \pi t$

$\frac{2\pi}{3} = \pi t$

$t = \frac{2}{3}$

2 -



a) $\uparrow$ $mg = 3mg\cos\alpha$

$\cos\alpha = \frac{1}{3}$

$\alpha = 70.5°$ (1dp)

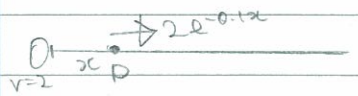
b) $\leftarrow [F = ma]$

$3mg\sin\alpha = m2gkr$

$3\sin\alpha = 2kl\sin\alpha$

$l = \frac{3}{2k}$

3 -



a) $\rightarrow [F = ma]$

$2e^{-0.1x} = 2.5v\frac{dv}{dx}$

$2\int_0^x e^{-0.1x}\,dx = 2.5\int_2^v v\,dv$

$\frac{-2}{0.1}\left[e^{-0.1x}\right]_0^x = 1.25\left[v^2\right]_2^v$

$-20e^{-0.1x} + 20 = 1.25v^2 - 5$

$v^2 = 20 - 16e^{-0.1x}$

b) $4^2 = 20 - 16e^{-0.1x}$

$16e^{-0.1x} = 4$

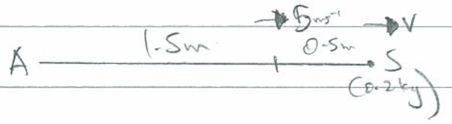
$e^{-0.1x} = \frac{1}{4}$

$-0.1x = \ln\frac{1}{4}$

$x = 10\ln 4$

c) As $x \to \infty$, $16e^{-0.1x} \to 0$

$\therefore v^2 \to 20$ and v cannot exceed $\sqrt{20}$ ms^{-1}

4. a) 

$$\frac{1}{2}\times0.2\times5^2=\frac{1}{2}\times0.2\times v^2+\frac{20\times0.5^2}{3}$$

$$2.5=0.1v^2+\frac{5}{3}$$

$$v^2=\frac{50}{6}$$

$$v=2.89\,ms^{-1}\ (3sf)$$

b) $$\frac{1}{2}(0.2)(5^2)=\frac{1}{2}(0.2)(1.5^2)+\frac{20x^2}{3}$$

$$2.5=0.225+\frac{20}{3}x^2$$

$$6.825=20x^2$$

$$x^2=0.34125$$

$$x=0.584m$$

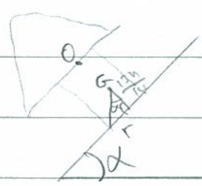
$$T=\frac{\lambda x}{a}=\frac{20\times0.584}{1.5}=7.79N\ (3sf)$$

5. a) $$-\frac{1}{3}\times\pi\times(2r)^2\times h\times\frac{h}{4}+\pi r^2h\times\frac{h}{2}=\left[\frac{1}{3}\pi(2r)^2h+\pi r^2h\right]OG$$

$$\frac{h}{2}-\frac{h}{3}=\frac{7}{3}OG$$

$$\frac{h}{6}=\frac{7}{3}OG$$

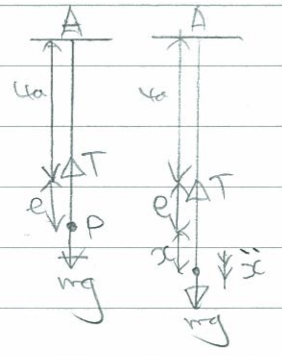
$$OG=\frac{3h}{6\cdot7}=\frac{1}{14}h$$

b) 

$$\tan\alpha=\frac{r}{\frac{13h}{14}}=r\times\frac{14}{13h}=\frac{7}{26}$$

$$r=\frac{7\times13h}{14\cdot26}=\frac{1}{4}h$$

6.



a) $T = mg \qquad T = \frac{\lambda x}{a}$

$$mg = \frac{8mge}{4a}$$

$$1 = \frac{8e}{4a}$$

$AO = 4a + \frac{1}{2}a = \frac{9}{2}a \qquad e = \frac{a}{2}$

b) $T = \frac{\lambda x}{a} = \frac{8mg(e+x)}{4a} = \frac{2mge}{a} + \frac{2mgx}{a}$

$\downarrow \boxed{F = ma}$

$mg - T = m\ddot{x}$

$$mg - \frac{2mge}{a} - \frac{2mgx}{a} = m\ddot{x}$$

$$g - \frac{2ga}{2a} - \frac{2gx}{a} = \ddot{x}$$

$$\ddot{x} = -\frac{2g}{a}x$$

$\therefore$ SHM with $\omega^2 = \frac{2g}{a}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{a}}} = \frac{2\pi\sqrt{a}}{\sqrt{2g}} = \frac{2\pi\sqrt{2a}}{2\sqrt{g}} = \pi\sqrt{\frac{2a}{g}}$$

c) $\frac{1}{2}\sqrt{ga} = r\omega$

$$\frac{1}{2}\sqrt{ga} = d \cdot \sqrt{\frac{2g}{a}}$$

$$d = \frac{\sqrt{ga}\cdot\sqrt{a}}{2\sqrt{2g}}$$

$$= \frac{a}{2\sqrt{2}}$$

$$= \frac{a\sqrt{2}}{4}$$

d) The particle moves up with SHM until it becomes slack, then moves freely under gravity, then it starts SHM again when it is taut again

7- a) $ME_A = ME_P$

$\frac{1}{2}\cancel{m}u^2 = \cancel{m}g(l - l\cos\alpha)$

$u^2 = 2gl - 2gl \cdot \frac{2}{3}$

$= \frac{2}{3}gl$

$u = \sqrt{\frac{2gl}{3}}$

b) $\frac{1}{2}\cancel{m}u^2 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(l - l\cos\theta)$

$u^2 = v^2 + 2gl - 2gl\cos\theta$

$\frac{2gl}{3} = v^2 + 2gl - 2gl\cos\theta$

$v^2 = 2gl\cos\theta - \frac{4}{3}gl$

$\boxed{F = ma}$

$T - mg\cos\theta = \frac{m}{\cancel{l}}(2g\cancel{l}\cos\theta - \frac{4}{3}g\cancel{l})$

$T = mg\cos\theta + 2mg\cos\theta - \frac{4}{3}mg$

$= 3mg\cos\theta - \frac{4}{3}mg = \frac{mg}{3}(9\cos\theta - 4)$

c) When $\cos\theta = \frac{2}{3}$, $T = \frac{mg}{3}(9 \cdot \frac{2}{3} - 4) = \frac{2mg}{3}$

When $\cos\theta = 1$, $T = \frac{mg}{3}(9-4) = \frac{5mg}{3}$

$\therefore \frac{2mg}{3} \leq T \leq \frac{5mg}{3}$