M3 - June 2001

1-a) $a = \frac{1}{2}e^{-\frac{1}{6}t}$

$\frac{dv}{dt} = \frac{1}{2}e^{-\frac{1}{6}t}$

$\int_{10}^{v} 1 \, dv = \frac{1}{2}\int_{0}^{t} e^{-\frac{1}{6}t} \, dt$

$[v]_{10}^{v} = -\frac{1}{2} \cdot 6\left[e^{-\frac{1}{6}t}\right]_{0}^{t}$

$v - 10 = -3e^{-\frac{1}{6}t} + 3$

$v = 13 - 3e^{-\frac{1}{6}t}$

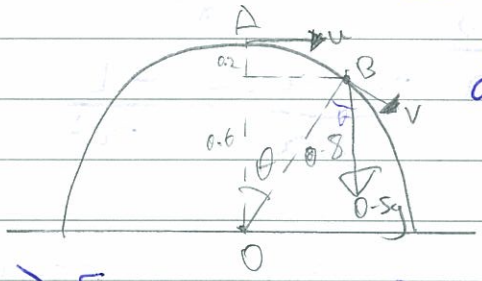
b) When $t = 3$

$v = 13 - 3e^{-\frac{1}{2}}$

$= 11.2 \text{ ms}^{-1}$ (3 sf)

c) $\lim_{x \to \infty} v = 13 \text{ ms}^{-1}$

2-



a) $\cos\theta = \frac{0.6}{0.8} = \frac{3}{4}$

b) [F = ma at B]

$mg\cos\theta = \frac{mv^2}{r}$

$0.5 \times g \times \frac{3}{4} = \frac{0.5v^2}{0.8 \times 0.2}$

$v^2 = 0.2 \times 3g$

$= 5.88 \text{ m}^2\text{s}^{-2}$

c) $ME_A = ME_B$

$\frac{1}{2}mu^2 + mg \cdot 0.2 = \frac{1}{2}mv^2$

$u^2 + 0.4g = 5.88$

$u^2 = 5.88 - 0.4g$

$= 1.96$

$u = 1.4 \text{ ms}^{-1}$

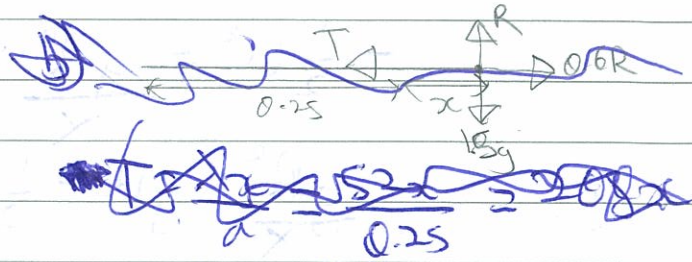
3- a) $ME_I = ME_{II}$

$\frac{1}{2}mv^2 = \frac{\lambda x^2}{2a}$

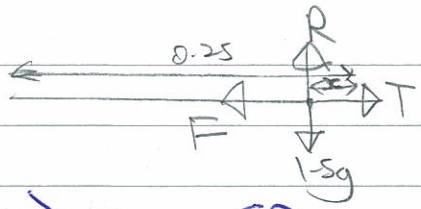
$\frac{1}{2} \times 1.5 \times v^2 = \frac{52 \times 0.05^2}{0.5}$

$v^2 = \frac{2 \times 0.26}{1.5}$

$v = 0.589 \text{ ms}^{-1}$ (3 sf)



b)



$\uparrow R = 1.5g$

$T = \frac{\lambda x}{a} = \frac{52x}{0.25} = 208x \quad \rightarrow T = F$

$208x = F$

In limiting eqm

$F \leq \mu R$

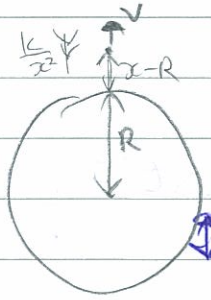
$F \leq 0.6R$

$F \leq 0.6 \times 1.5g$

$208x \leq 0.9g$

$x \leq 0.042\text{m}$ ∴ min distance $= 0.25 - 0.042 = 0.208\text{m}$

4-



a) ~~$F = ma$~~ ~~$-mg = m\frac{dv}{dt}$~~

$a = -\frac{k}{x^2}$

$\frac{v\,dv}{dx} = \frac{-k}{x^2}$

$F = ma$

$+mg = \frac{mk}{R^2}$

$k = gR^2$

$\frac{v\,dv}{dx} = \frac{-gR^3}{x^2}$

b) $\frac{v\,dv}{dx} = -\frac{gR^3}{x^2}$

$\int_u^0 v\,dv = -gR^2 \int_R^X \frac{1}{x^2}\,dx$

$\frac{1}{2}\left[v^2\right]_u^0 = -gR^2\left[-x^{-1}\right]_R^X$

$-\frac{1}{2}u^2 = \frac{gR^2}{X} - \frac{gR^2}{R}$

$\frac{gR^2}{X} = gR - \frac{1}{2}u^2$

$X = \frac{2gR^2}{2gR - u^2}$

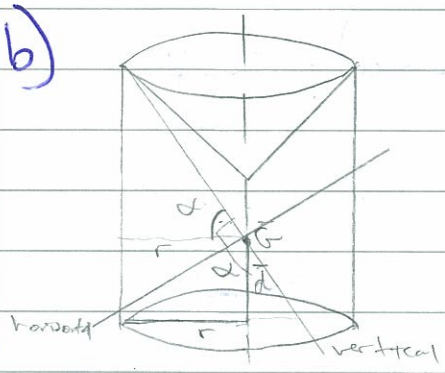
5. a) $\pi r^2 h \cdot \frac{h}{2} - \frac{1}{3}\pi r^2 h \cdot \frac{1}{2}\left(\frac{h}{2} + \frac{3}{4}\cdot\frac{1}{2}h\right) = \left(\pi r^2 h - \frac{1}{3}\pi r^2 \frac{h}{2}\right)\bar{x}$

$6h - 2\left(\frac{7h}{8}\right) = (12-2)\bar{x}$

$10\bar{x} = \frac{17}{4}h$

$\bar{x} = \frac{17}{40}h$

b)

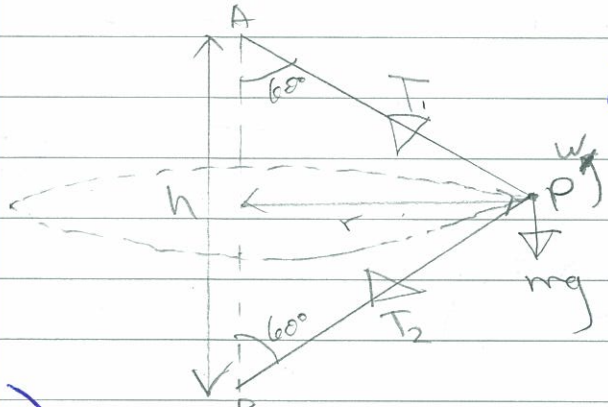


$\tan\alpha = \left(h - \frac{17}{40}h\right) \div r$

$= \frac{23h}{40r} = \frac{4\times 23r}{40r} = \frac{23}{10}$

$\alpha = 66.5°\ (1dp)$

6.



a) $\tan 60 = \frac{r}{\frac{1}{2}h}$

$r = \frac{1}{2}h\tan 60 = \frac{\sqrt{3}}{2}h$

b) $\uparrow T_1\cos 60 - mg - T_2\cos 60 = 0$

$T_1 - T_2 = 2mg$ ①

$\overrightarrow{F=ma}$

$T_1\sin 60 + T_2\sin 60 = mr\omega^2$

$\frac{\sqrt{3}}{2}T_1 + \frac{\sqrt{3}}{2}T_2 = mr\omega^2 \cdot \frac{\sqrt{3}}{2}$

$T_1 = mr\omega^2 - T_2$

$mr\omega^2 - T_2 - T_2 = 2mg$

$T_2 = \frac{mr\omega^2 - 2mg}{2}$

$T_1 = 2mg + \frac{mr\omega^2 - 2mg}{2} - mg = \frac{mr\omega^2 + 2mg}{2}$

c) $T_2 = \frac{mr\omega^2 - 2mg}{2}$

$\frac{mr\omega^2 - 2mg}{2} > 0$

$mr\omega^2 > 2mg$

$\omega^2 > \frac{2g}{r}$

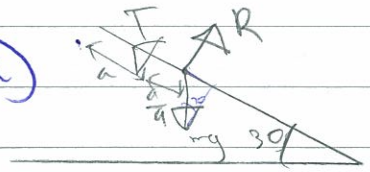
$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T}$

$\frac{2\pi}{T} < \sqrt{\frac{2g}{r}}$

$T < \frac{2\pi\sqrt{r}}{\sqrt{2g}}$

$T < \pi\sqrt{\frac{2r}{g}}$

7- a) 

$T = mg\sin 30 = \frac{1}{2}mg$

$T = \frac{\lambda x}{a} = \frac{\lambda \frac{a}{8}}{a} = \frac{\lambda}{8}$

$\frac{mg}{2} = \frac{\lambda}{8}$

$\lambda = 4mg$

b) $T = \frac{\lambda x}{a} = \frac{4mg\left(\frac{1}{8}a + x\right)}{a} = \frac{1}{2}mg + \frac{4mgx}{a}$

$\boxed{F = ma}$

$-T + mg\sin 30 = m\ddot{x}$

$-\frac{1}{2}mg - \frac{4mx}{a} + mg\times\frac{1}{2} = m\ddot{x}$

$\ddot{x} = -\frac{4g}{a}x$

$\omega^2 = \frac{4g}{a}$

$\omega = 2\sqrt{\frac{g}{a}}$

$T = \frac{2\pi}{\omega} = 2\pi \times \frac{\sqrt{a}}{2\sqrt{g}} = \pi\sqrt{\frac{a}{g}}$

c) $a_{max} = \omega^2 r = \frac{4g}{a} \times \frac{1}{4}a = g$

d) $x = a\sin\omega t$

$x = \frac{1}{4}a\sin\left(2\sqrt{\frac{g}{a}}t\right)$

$+\frac{a}{8} = \frac{1}{4}a\sin\left(2\sqrt{\frac{g}{a}}t\right)$

$\sin\left(2\sqrt{\frac{g}{a}}t\right) = +\frac{1}{2}$

$2\sqrt{\frac{g}{a}}t = \frac{\pi}{6}$

$t = \frac{\pi}{12}\sqrt{\frac{a}{g}}$