M3 January 2013 (MA)

Q1) $a = 9x$

$v\frac{dv}{dx} = 9x$

$\int (v)\,dv = 9\int (x)\,dx$

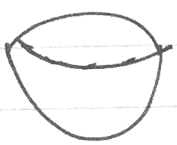
$\frac{v^2}{2} = \frac{9x^2}{2} + c$

$x = 2,\ v = 6$: $18 = 9(2) + c$

$c = 0$ //

$\therefore \boxed{v^2 = 9x^2}$

Q2a)

Shape	Mass (volume)	↑+ Displacement of c.o.m from O
	$\frac{1}{3}\pi r^2(kr) = \boxed{\frac{\pi k r^3}{3}}$	$\boxed{\frac{kr}{4}}$
	$\boxed{\frac{2}{3}\pi r^3}$	$\boxed{\frac{-3r}{8}}$
	$\boxed{\frac{\pi r^3}{3}(2+k)}$	$\bar{y}$

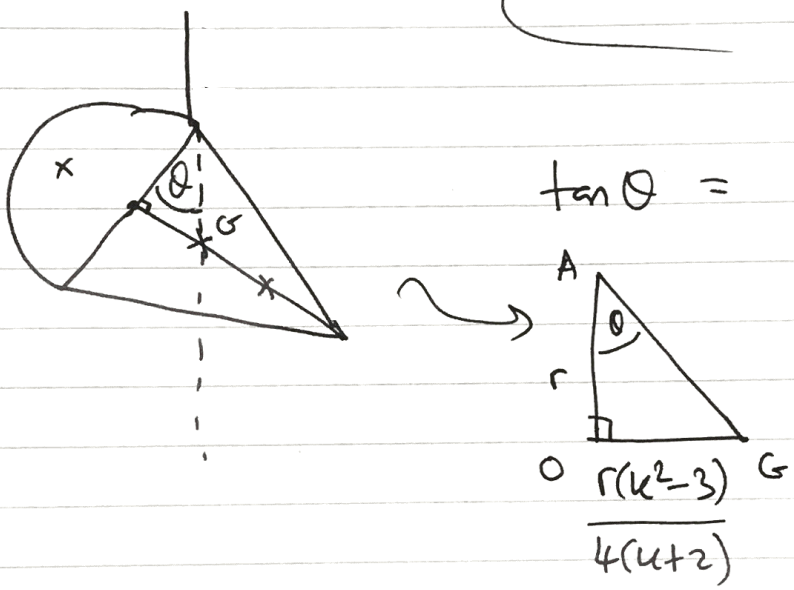
taking moments about a diameter through O...

$$\frac{\pi k r^3}{3}\left(\frac{kr}{4}\right) + \frac{2}{3}\pi r^3\left(\frac{-3r}{8}\right) = \frac{\pi r^3}{3}(2+k)(\bar{y})$$

$$\frac{k^3 r}{4} - \frac{3r}{4} = (k+2)\bar{y} = \frac{r(k^2-3)}{4}$$

$$\frac{r(k^2-3)}{4(k+2)} = \bar{y}$$

b)



$$\tan\theta = \frac{r(k^2-3)}{4r(k+2)} = \frac{11}{14}$$

so $\frac{11}{14} = \frac{k^2-3}{4(k+2)}$

$$k^2 - 3 = \frac{11}{14} \times 4(k+2)$$

$$k^2 - 3 = \frac{44}{14}k + \frac{44}{7}$$

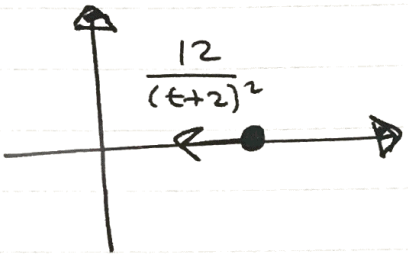
$$k^2 - \frac{44}{14}k - \frac{65}{7} = 0$$

$$7k^2 - 22k - 65 = 0$$

$$(7k + 13)(k - 5) = 0$$

$$\boxed{k = 5} \qquad (k > \sqrt{3})$$

(Q3a)



$$F = \frac{-12}{(t+2)^2} = 0.6a$$

$$\frac{-12}{(t+2)^2} = 0.6\frac{dv}{dt}$$

$$\int (1)\,dv = -20\int [(t+2)^{-2}]\,dt$$

$$v = -20\left[-(t+2)^{-1}\right] + c$$

$$v = \frac{20}{t+2} + c$$

$v = 15,\ t = 0$: $15 = 10 + c$

$\therefore c = 5$ //

So $v = \frac{20}{t+2} + 5 = 5\left(\frac{4}{t+2} + 1\right)$

b) $\frac{dx}{dt} = 5\left(\frac{4}{t+2} + 1\right)$

$\int (1)\, dx = 5\int \left[\frac{4}{t+2} + 1\right] dt$

$x = 5\left[4\ln(t+2) + t\right] + c$

$x = 0, t = 0$: $0 = 20\ln 2 + c$

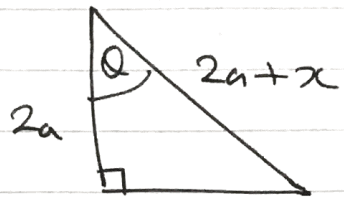
$\therefore c = -20\ln 2 = 20\ln\frac{1}{2}$.

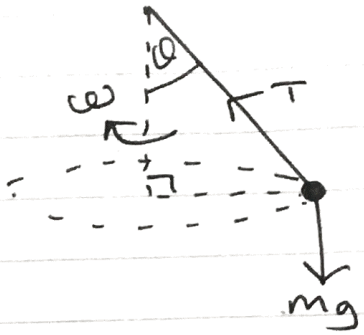
$\therefore x = 20\ln(t+2) + 5t + 20\ln\frac{1}{2}$.

$t = 5$: $x = 20\ln 7 + 25 + 20\ln\frac{1}{2}$

$= \boxed{25 + 20\ln\frac{7}{2}} = 50.1$

Q4a) $R(\updownarrow)$: $T\cos\theta = mg$





$\cos\theta = \frac{2a}{2a+x}$ $\therefore T\left(\frac{2a}{2a+x}\right) = mg$ //

but $T = \frac{\lambda x}{L} = \frac{6mgx}{2a} = \frac{3mgx}{a}$.

so $\frac{3m\cancel{g}x}{\cancel{a}}\left(\frac{2\cancel{a}}{2a+x}\right) = m\cancel{g}$

$\Rightarrow 3x\left(\frac{2}{2a+x}\right) = 1$

$\Rightarrow 6x = 2a + x$

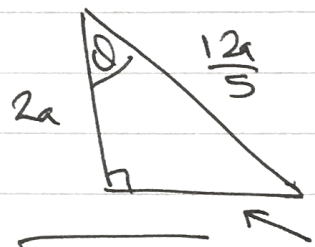
$\Rightarrow 5x = 2a$ $\therefore \boxed{x = \frac{2a}{5}}$

b) $\overset{+}{\leftarrow}$ N2L (P): $T\sin\theta = \frac{mv^2}{r}$

$\left(\frac{3mg}{a}\left(\frac{2a}{5}\right)\right)\left(\frac{\sqrt{11}}{6}\right) = \frac{M}{r}v^2$

$\frac{6mg}{5}\left(\frac{\sqrt{11}}{6}\right) = \frac{mv^2}{\frac{2a\sqrt{11}}{5}}$

$\therefore v^2 = \frac{g\sqrt{11}}{5} \times \frac{2a\sqrt{11}}{5} = \boxed{\frac{22ag}{25}}$

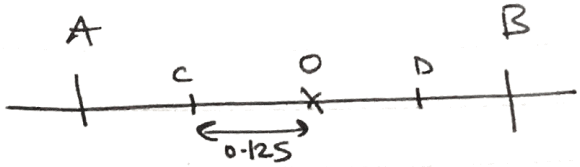


$\sqrt{\left(\frac{12}{5}a\right)^2 - (2a)^2} = \frac{2a\sqrt{11}}{5}$ //

$\therefore \sin\theta = \frac{2\sqrt{11}}{\frac{12}{5}}$

$\sin\theta = \frac{\sqrt{11}}{6}$

Q5a)



$a = 0.25$

$x = 0.25 \sin \omega t$.

from O to C : $0.125 = 0.25 \sin \omega t$

$\frac{1}{2} = \sin(0.1\omega)$

from C to D takes 0.2s.
So from O to C should take 0.1s

$\Rightarrow \quad 0.1\omega = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\Rightarrow \quad \therefore \omega = \frac{\pi}{6 \times 0.1} = \frac{10\pi}{6}$ //

$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{10}{6}\pi} = \boxed{\frac{6}{5}}$

b) P is moving towards A initially. so this dir. is +ve.

$x = 0.25 \sin \omega t = 0.25 \sin\left(2 \times \frac{10\pi}{6}\right)$

$= -0.2165...$

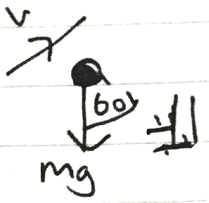
so distance from B $= -0.2165 + 0.25$

$= \boxed{0.033\text{m}}$

c) $a_{max} = a\omega^2 = 0.25 \times \left(\frac{5\pi}{3}\right)^2 = \boxed{6.85 \text{ ms}^{-2}}$

d) $V_{max} = a\omega = 0.25 \times \frac{5\pi}{3} = \boxed{1.31 \text{ ms}^{-2}}$

Q6a) <u>at B</u> : $mg\cos 60 + R = \frac{mv^2}{a}$



but $R = 0$ as P loses contact

so $\frac{mg}{2} = \frac{mv^2}{a}$

$v^2 = \frac{ag}{2} \quad \therefore v = \sqrt{\frac{ag}{2}}$

b) <u>$\Delta KE = \Delta GPE$ from A → B</u>

$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga\sin 30$

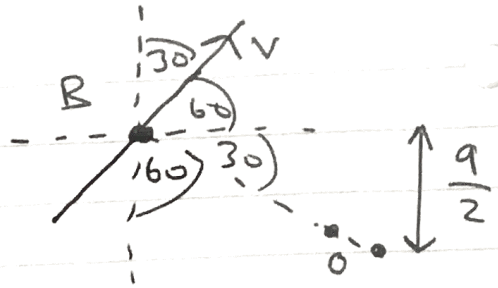
$\frac{u^2}{2} - \frac{v^2}{2} = \frac{ag}{2}$

$\therefore u^2 = ag + v^2$

$u^2 = ag + \frac{ag}{2} = \frac{3ag}{2}$

$\therefore \boxed{u = \sqrt{\frac{3ag}{2}}}$

c)



$+\uparrow$

$s = -\frac{9}{2}$

$u = V\cos 30 = \sqrt{\frac{9g}{2}}\cos 30$

$v = v$

$a = -g$

$t =$

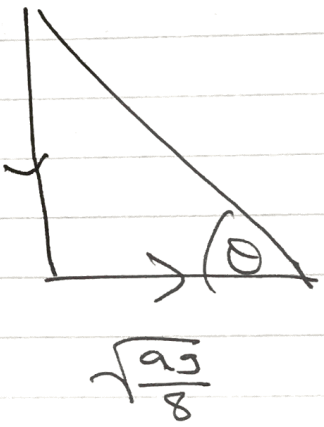
$s = ut + \frac{1}{2}at^2$ $\qquad v = u + at$

$-\frac{9}{2} =$ $\qquad V = \sqrt{\frac{9g}{2}}\cos 30$

$v^2 = u^2 + 2as$

$v^2 = \frac{9g}{2}\cos^2 30 + \frac{2g9}{2} = \frac{11g9}{8}$ //

horizontal speed $= \sqrt{\frac{9g}{2}}\cos 60 = \sqrt{\frac{9g}{8}}$ //.

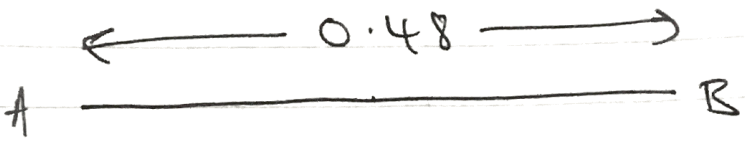


$\tan\theta = \frac{\sqrt{\frac{11}{8}}}{\sqrt{\frac{1}{8}}}$

$= \sqrt{11}$ //

$\therefore \theta = \tan^{-1}\sqrt{11} = \boxed{73^\circ}$ to 2sf.

Q7a)

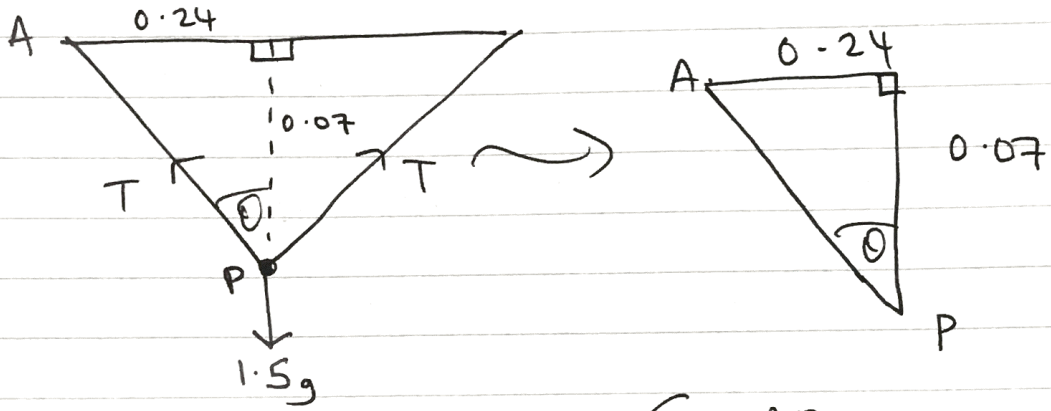


$$T = \frac{\lambda x}{l} = 240$$

$$\frac{\lambda(0.48 - 0.30)}{0.3} = 240$$

$$\therefore \lambda = \frac{240 \times 0.3}{0.18} = \boxed{400\text{N}}$$

b)



$\therefore AP = 0.25$ (Pythagorean triplet)

$\therefore \cos\theta = \frac{0.07}{0.25} = \frac{7}{25}$

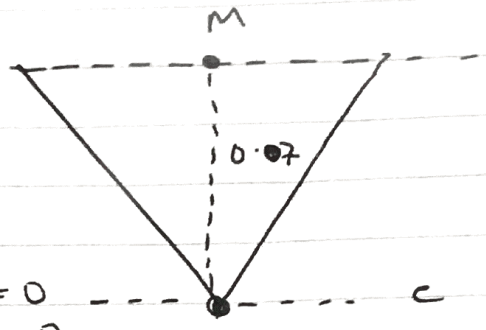
$+\uparrow$ N2L(P): $2T\cos\theta - 1.5g = 1.5a$.

$$T = \frac{\lambda x}{l} = \frac{400(0.5 - 0.3)}{0.3}$$

$$T = \frac{800}{3} \rightarrow \frac{2\left(\frac{800}{3}\right)\left(\frac{7}{25}\right) - 1.5(9.8)}{1.5} = a$$

$$\Rightarrow a = \boxed{89.8\,\text{ms}^{-2}}$$

c)



$KE = \frac{1}{2}(1.5)v^2$

$GPE = 1.5g(0.07)$

$EPE = \frac{400(0.18)^2}{2(0.3)}$

$KE = 0$

$GPE = 0$

$EPE = \frac{400(0.5-0.3)^2}{2(0.3)} = \frac{80}{3}$

C.O.E : $0.75v^2 + 1.5g(0.07) + \frac{400(0.18)^2}{2(0.3)} = \frac{80}{3}$

$\Rightarrow 0.75v^2 = \frac{80}{3} - 1.5g(0.07) - 21.6$

$\Rightarrow 0.75v^2 = 4.037667..$

$\Rightarrow v^2 = \frac{4.03766..}{0.75} = 5.3836..$

$\Rightarrow$ **$v = 2.3$ m/s**