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M3 January 2012 (MA)

Q1)

Before

$KE = 0$

$GPE = 0.8g(1.1)$

$EPE = 0$

After

$KE = 0$

$GPE = 0$

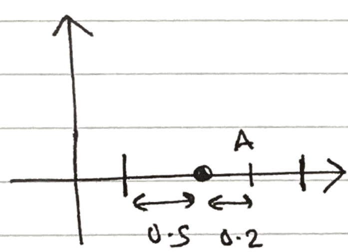
$EPE = \dfrac{\lambda(1.1-0.6)^2}{2(0.6)}$

$\underline{\Delta GPE = \Delta EPE}$

$\Rightarrow \quad 0.8g(1.1) = \dfrac{\lambda(0.5)^2}{1.2}$

$\Rightarrow \quad \lambda = \dfrac{0.8g(1.1)}{\dfrac{0.5^2}{1.2}} = \boxed{41.4\text{N}}$

Q2a)



$T = \dfrac{2\pi}{3} = \dfrac{2\pi}{\omega} \quad \therefore \omega = 3$

$a = -\omega^2 x = -(9)(0.2)$

$= -1.8$

$\therefore |a| = \boxed{1.8\text{ms}^{-2} \text{ towards } C}$

b) $v^2 = \omega^2(a^2 - x^2) = 9(0.5^2 - 0.2^2)$

$= 1.89 \quad \therefore v = \sqrt{1.89} = \boxed{1.37\text{ms}^{-1}}$

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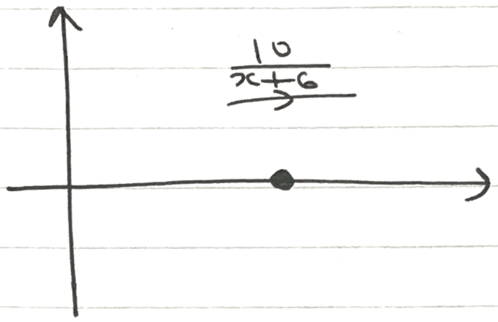
$a = 0.5 \qquad \omega = 3.$

c) $x = 0.5\sin(3t) = 0.2$

$\therefore \sin^{-1}\left(\frac{0.2}{0.5}\right) = 3t$

$\therefore t = \frac{1}{3}\sin^{-1}\left(\frac{2}{5}\right) = \boxed{0.1375}$

Q3a)



$v = 10(x+6)^{-1}$

$a = \frac{dv}{dx} \times v$ //

$\therefore a = -10(x+6)^{-2} \times v$

$a = \frac{-10v}{(x+6)^2}$

at $x = 14$, $v = \frac{10}{20} = \frac{1}{2}$.

$\therefore a = \frac{-5}{(x+6)^2} = \frac{-5}{(14+6)^2}$ at $x = 14$

$\Rightarrow a = \frac{-5}{20^2} = \boxed{-\frac{1}{80}\ ms^{-2}}$

b) $V = 10(x+6)^{-1}$

$\frac{dx}{dt} = \frac{10}{x+6}$

$(x+6)\frac{dx}{dt} = 10$

$\int (x+6)\,dx = \int (10)\,dt$

$\frac{x^2}{2} + 6x = 10t + c$

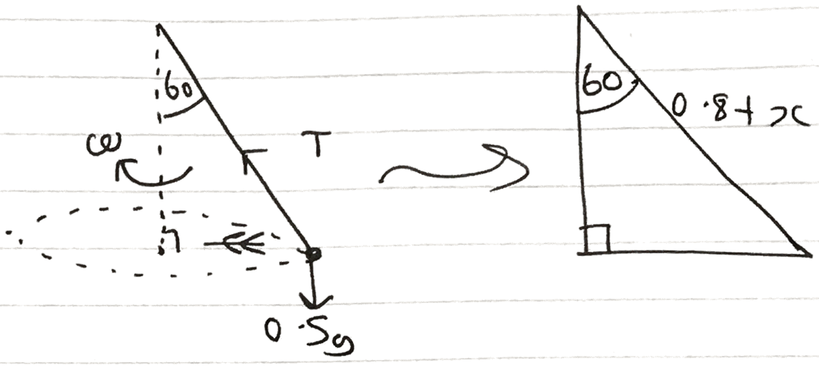
$\underline{x=2,\ t=1}$: $2 + 12 = 10 + c$

$\therefore c = 4$

$\Rightarrow \frac{x^2}{2} + 6x = 10t + 4$

$\underline{x=14}$: $\dfrac{\frac{14^2}{2} + 6(14) - 4}{10} = t = \boxed{17.8s}$

Q4a)



$R(\updownarrow)$: $T\cos 60 = 0.5g$

$$\frac{19.6x}{0.8} = \frac{0.5g}{\cos 60}$$

$$x = \frac{(0.5g)(0.8)}{19.6\cos 60} = \frac{2}{5} = \boxed{0.4m}$$

b) $\sin 60 = \dfrac{r}{0.8+0.4}$ $\quad \therefore r = 1.2\sin 60$

$\overleftarrow{N2L(P)}$: $T\sin 60 = 0.5(1.2\sin 60)(\omega^2)$

$$\omega^2 = \frac{T\cancel{\sin 60}}{0.6\cancel{\sin 60}} = \frac{T}{0.6}$$

$$T = \frac{\lambda x}{l} = \frac{19.6(0.4)}{0.8} = 9.8$$

$$\therefore \omega^2 = \frac{9.8}{0.6}$$

$$\omega = \sqrt{\frac{9.8}{0.6}} = \boxed{4.04}$$

Q5a) Distance from centre of earth $= R + x$

$$\therefore F = \frac{k}{(R+x)^2}$$

at $x=0$, $F=mg$: $\quad mg = \frac{k}{R^2}$

$$\therefore k = mgR^2$$

so $F = \frac{mgR^2}{(R+x)^2}$ $\square$

b) $$\frac{-\cancel{m}gR^2}{(R+x)^2} = \cancel{m}a = \cancel{m}\frac{dv}{dx}(v)$$

$$\frac{-gR^2}{(R+x)^2} = v\frac{dv}{dx}$$

$$\int (v)\,dv = -gR^2\int \left((R+x)^{-2}\right) dx$$

$$\frac{v^2}{2} = -gR^2\left[-(R+x)^{-1}\right] + c$$

$$\frac{v^2}{2} = \frac{gR^2}{R+x} + c$$

$x = 2R$, $v = \sqrt{\frac{gR}{2}}$: $\quad \frac{gR}{4} = \frac{gR^{\cancel{2}}}{3\cancel{R}} + c$

$c = \frac{-gR}{12}$ //

$\therefore \frac{v^2}{2} = \frac{gR^2}{R+x} - \frac{gR}{12}$.

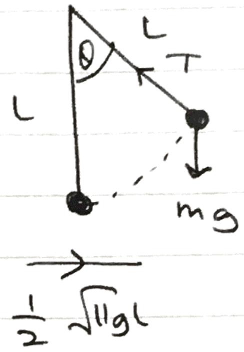
$\underline{x = R}$: $\frac{v^2}{2} = \frac{gR^{\cancel{2}}}{2\cancel{R}} - \frac{gR}{12}$

$\frac{v^2}{2} = \frac{gR}{2} - \frac{gR}{12} = \frac{5gR}{12}$

$\therefore v^2 = \frac{5gR}{6}$

$\therefore \boxed{v = \sqrt{\frac{5gR}{6}}}$

Q6a) using conservation of energy to find speed when P has turned through an angle θ,



__Initially__ : $KE = \frac{1}{2}m\left(\frac{11gl}{4}\right)$

$GPE = 0$

__Finally__ : $KE = \frac{1}{2}mv^2$

$GPE = mgl(1-\cos\theta)$.

$$\Rightarrow \quad \frac{11\cancel{m}gl}{8} = \frac{\cancel{m}v^2}{2} + \cancel{m}gl(1-\cos\theta)$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{11gl}{8} - gl + gl\cos\theta$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{3gl}{8} + gl\cos\theta$$

$$\Rightarrow \quad v^2 = \frac{3gl}{4} + 2gl\cos\theta \quad //$$

__N2L(P)__ $\nwarrow^{+}$: $T - mg\cos\theta = \frac{m}{l}v^2$

$$T - mg\cos\theta = \frac{m}{l}\left(\frac{3gl}{4} + 2gl\cos\theta\right)$$

$$T - mg\cos\theta = \frac{3mg}{4} + 2mg\cos\theta$$

$$T = \frac{3mg}{4} + 3mg\cos\theta$$

$$T = 3mg\left(\cos\theta + \frac{1}{4}\right)$$

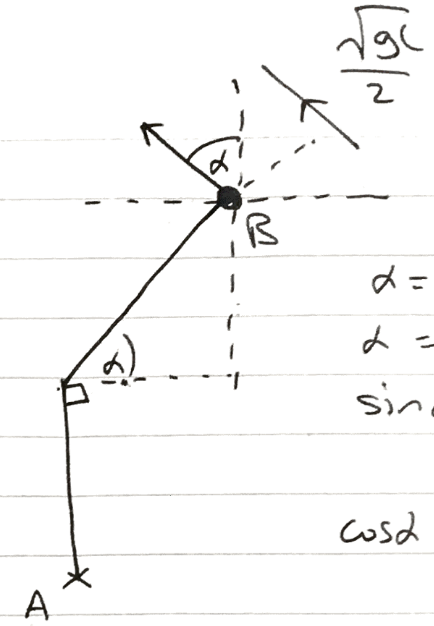
b) $\underline{T=0}$: $\cos\theta = -\frac{1}{4}$

$$\left(\cos^{-1}\left(-\frac{1}{4}\right) = 104.48°\right)$$

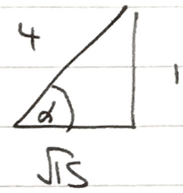
(from a) : $v^2 = \frac{3gl}{4} + 2gl\left(-\frac{1}{4}\right) = \frac{gl}{4}$

$$\therefore v = \sqrt{\frac{gl}{4}} = \boxed{\frac{\sqrt{gl}}{2}}$$

c) $\uparrow +$
$S = s$
$u = \frac{\sqrt{gl}}{2}\cos\alpha$
$V = 0$
$a = -g$
$t =$



$\alpha = \theta - 90$
$\alpha = 14.48...^\circ$
$\sin\alpha = \frac{1}{4}$ //
$\cos\alpha = \frac{\sqrt{15}}{4}$



$$V^2 = u^2 + 2as$$

$$0 = \frac{gl}{4}\left(\frac{\sqrt{15}}{4}\right)^2 - 2gs$$

$$2gs = \frac{15gl}{64}$$

$$\therefore s = \frac{15\cancel{g}l}{128\cancel{g}} = \boxed{\frac{15l}{128}} //$$

Q7a) $V = \pi\int_2^6 y^2\,dx = \frac{\pi}{4}\int_2^6 x^2(6-x)^2\,dx$

$$= \frac{\pi}{4}\int_2^6 \left[x^2(36 - 12x + x^2)\right] dx$$

$$= \frac{\pi}{4}\int_2^6 \left[36x^2 - 12x^3 + x^4\right] dx$$

$$= \frac{\pi}{4}\left[12x^3 - 3x^4 + \frac{x^5}{5}\right]_2^6$$

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$$= \frac{\pi}{4}\left[\frac{1296}{5}\right] - \frac{\pi}{4}\left[\frac{272}{5}\right] = \frac{256\pi}{5}$$

$$M\bar{x} = p\pi\int_2^6 y^2 x \, dx = \frac{p\pi}{4}\int_2^6 \left[36x^3 - 12x^4 + x^5\right] dx$$

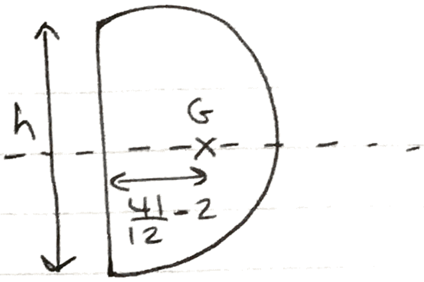
$$= \frac{p\pi}{4}\left[\frac{36x^4}{4} - \frac{12x^5}{5} + \frac{x^6}{6}\right]_2^6$$

$$= \frac{p\pi}{4}\left[\frac{3888}{5}\right] - \frac{p\pi}{4}\left[\frac{1168}{15}\right] = \frac{10496p\pi}{15\times 4}$$

and $M = \dfrac{256p\pi}{5}$

$$\therefore \bar{x} = \frac{\frac{10496p\pi}{15\times4}}{\frac{256p\pi}{5}} = \boxed{\frac{41}{12}}$$

b) once rotated through 360°...
we get a 'shape' like this:

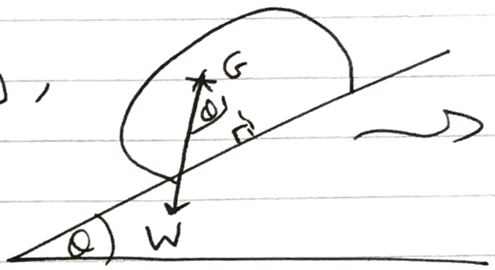


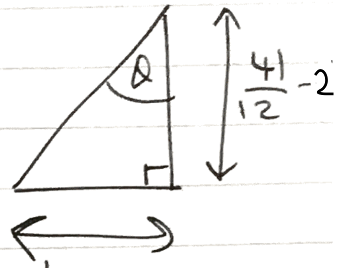
to find h : $x=2$: $y=4$

$\therefore h = 2\times4 = 8$

Now on the plane:

at point of toppling, weight will act through the point of lowest contact with the plane.



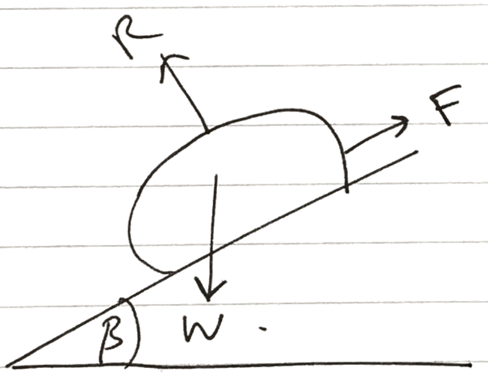


$\frac{h}{2} = 4$

$\tan\theta = \dfrac{4}{\frac{41}{12}-2} = \dfrac{48}{17}$

$\alpha = \theta = \tan^{-1}\left(\dfrac{48}{17}\right) = \boxed{70.5°}$

c)



$R \nearrow^{+}$: $F = W\sin\beta$.

$R \nwarrow^{+}$: $R = W\cos\beta$.

and $F = 0.3R$

$\Rightarrow \cancel{W}\sin\beta = 0.3\cancel{W}\cos\beta$

$\Rightarrow \tan\beta = 0.3 \quad \therefore \beta = \tan^{-1}(0.3)$
$= \boxed{16.7°}$