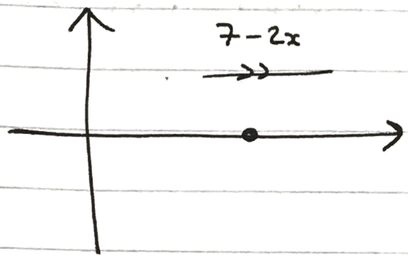
M3 January 2011 (MA)

Q1)



$$a = 7 - 2x$$

$$v\frac{dv}{dx} = 7 - 2x$$

$$\int (v)\,dv = \int (7-2x)\,dx$$

$$\frac{v^2}{2} = 7x - x^2 + c$$

$\underline{x = 0,\ v = 6}$: $18 = c$ //

$$\therefore \frac{v^2}{2} = 7x - x^2 + 18$$

$\underline{v = 0}$: $0 = 7x - x^2 + 18$

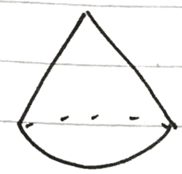
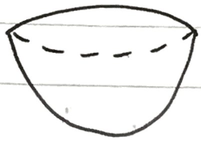
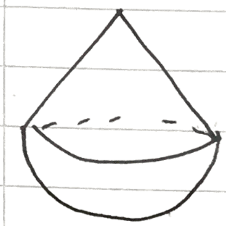
$$x^2 - 7x - 18 = 0$$

$$(x+2)(x-9) = 0$$

$$\boxed{x = 9}$$

P cant be on the negative x-axis when ~~it first~~ comes to rest

(Q2a)

Shape	Mass (vol.)	$\uparrow^{+}$ Displacement of c.o.m from O
	$4m$	$\frac{3r}{8}$
	km	$-\frac{r}{2}$
	$(4+k)m$	0

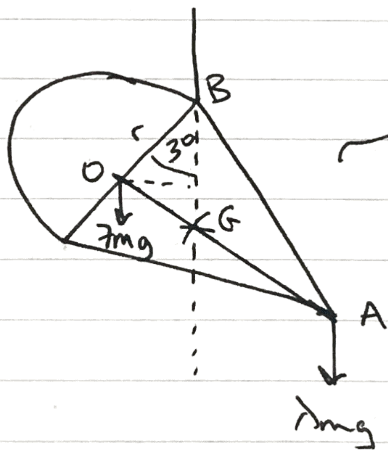
taking moments about a diameter through O . . .

$$4\cancel{m}\cancel{g}\left(\frac{3r}{8}\right) - k\cancel{m}\cancel{g}\left(\frac{r}{2}\right) = (4+k)\cancel{m}\cancel{g}\,(0)$$

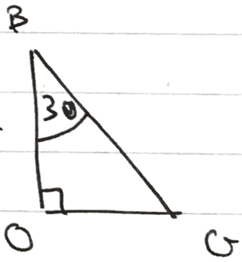
$$\frac{3\cancel{r}}{2} = \frac{\cancel{r}k}{2}$$

$$\therefore \boxed{k=3}$$

b)



ΔOBG : r



$$\text{so } \tan 30 = \frac{OG}{r}$$

$$\therefore OG = r\tan 30 \quad //$$

so if $OG = r\tan 30$
then $AG = 2r - r\tan 30$ //
(as $OG + GA = 2r$)

now taking moments about the downward vertical,

$$\lambda \cancel{m}\cancel{g}\cos 30(2r - r\tan 30) = 7\cancel{m}\cancel{g}\cos 30\,(r\tan 30)$$

$$\lambda \cancel{r}(2 - \tan 30) = 7\cancel{r}(\tan 30)$$

$$\lambda = \frac{7\tan 30}{2 - \tan 30} \simeq \boxed{2.84}$$

Q3a) $V = \pi\int_1^2 y^2\,dx = \pi\int_1^2 (e^{2x})\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_1^2$

$$= \pi\left[\frac{1}{2}e^4\right] - \pi\left[\frac{1}{2}e^2\right] = \frac{\pi}{2}(e^4 - e^2)$$

b) $M\bar{x} = \rho\int_1^2 \pi y^2 x\,dx$

$$M\bar{x} = \rho\pi\int_1^2 [e^{2x} \times x]\,dx = \rho\pi\int_1^2 (xe^{2x})\,dx$$

<u>By parts</u> : $\frac{dv}{dx} = e^{2x} \longrightarrow v = \frac{1}{2}e^{2x}$

$$u = x \longrightarrow u' = 1$$

$$\therefore M\bar{x} = \rho\pi\left[\left[\frac{1}{2}xe^{2x}\right]_1^2 - \frac{1}{2}\int_1^2 (e^{2x})\,dx\right]$$

$$= \rho\pi\left[\left[e^{2\cdot 2}\right] - \left[\frac{1}{2}e^2\right] - \left[\frac{1}{4}e^{2x}\right]_1^2\right]$$

$$= \rho\pi\left[e^4 - \frac{1}{2}e^2 - \left[\frac{1}{4}e^4 - \frac{1}{4}e^2\right]\right]$$

$$= \left[e^4 - \frac{1}{4}e^4 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right] \times p\pi$$

$$= \left[\frac{3}{4}e^4 - \frac{1}{4}e^2\right] \times p\pi$$

$$= \frac{p\pi e^2}{4}\left[3e^2 - 1\right] = M\bar{x} = \frac{p\pi}{4}(3e^4 - e^2)$$

and $M = \frac{p\pi}{2}(e^4 - e^2)$

$$\therefore \bar{x} = \frac{\frac{\cancel{p\pi}}{4}(3e^4 - e^2)}{\frac{\cancel{p\pi}}{2}(e^4 - e^2)} = \frac{1}{2} \times \frac{3e^2 - 1}{e^2 - 1}$$

$$= \boxed{1.66}$$

Q4a) $x = 5\sin\left(\frac{\pi t}{3}\right)$

$$\dot{x} = \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$$

$$\ddot{x} = -5\left(\frac{\pi^2}{9}\right)\sin\left(\frac{\pi t}{3}\right)$$

but $x = 5\sin\left(\frac{\pi t}{3}\right)$, $\ddot{x} = -\frac{\pi^2}{9} \times \left(5\sin\frac{\pi t}{3}\right)$

so $\ddot{x} = \frac{-\pi^2}{9}x$

hence P moves with S.H.M. (in the form $a = -\omega^2 x$)

b) $\ddot{x} = \frac{-\pi^2}{9}x$

$\omega^2 = \frac{\pi^2}{9} \quad \therefore \omega = \frac{\pi}{3} \quad \therefore T = \frac{2\pi}{\frac{\pi}{3}} = \boxed{6}\ s.$

amplitude $= \boxed{5}$ (as $x = 5\sin\frac{\pi t}{3} \rightarrow x_{max} = 5$).

c) $V_{max} = a\omega = 5 \times \frac{\pi}{3} = \boxed{\frac{5\pi}{3}\ m/s}$

d) $x = 5\sin\frac{\pi t}{3}$

$\underline{x = 2}$: $\frac{2}{5} = \sin\frac{\pi t}{3}$

$\therefore \frac{\pi t}{3} = \sin^{-1}\left(\frac{2}{5}\right) = 0.4115\ldots$

$t_A = \frac{3(0.4115\ldots)}{\pi} = 0.39297\ldots$

$\underline{x = 3}$: $\frac{3}{5} = \sin\frac{\pi t}{3}$

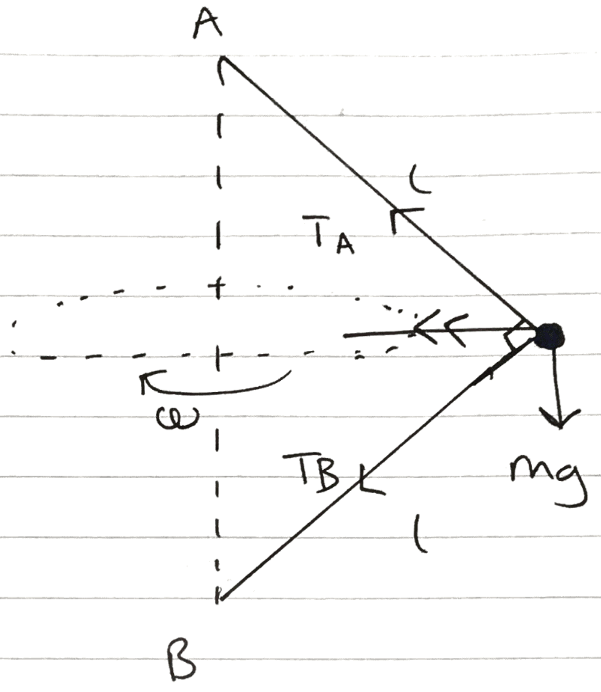
$\sin^{-1}\left(\frac{3}{5}\right) = \frac{\pi t}{3} = 0.64350\ldots$

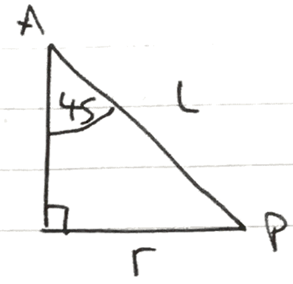
$\therefore t_B = \frac{3(0.64350\ldots)}{\pi} = 0.61450\ldots$

required time $= t_B - t_A = 0.61450\ldots - 0.39297\ldots$

$= \boxed{0.22s}$ to 2 s.f.

Q5a)





$r = l\sin 45 = \frac{l\sqrt{2}}{2}$

$R(\updownarrow)$: $T_A \cos 45 = mg + T_B \cos 45$

$\therefore \quad T_A - T_B = \frac{mg}{\cos 45} \sim (1)$

$R(\leftrightarrow^+)$: $T_A \sin 45 + T_B \sin 45 = m(r)(\omega)^2$

$T_A + T_B = \frac{\frac{ml\sqrt{2}}{2}\omega^2}{\sin 45}$

$\therefore \quad T_A + T_B = ml\omega^2 \sim (2)$

$(1) + (2)$: $2T_A = ml\omega^2 + mg\sqrt{2}$

$T_A = \frac{m}{2}(l\omega^2 + g\sqrt{2})$

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and $T_B = ml\omega^2 - T_A$

$$= ml\omega^2 - \frac{ml\omega^2}{2} - \frac{mg\sqrt{2}}{2}$$

$$= \frac{ml\omega^2}{2} - \frac{mg\sqrt{2}}{2}$$

$$\therefore T_B = \frac{m}{2}\left(l\omega^2 - g\sqrt{2}\right)$$

b) $\underline{T_B > 0}$: $\frac{m}{2}\left(l\omega^2 - g\sqrt{2}\right) > 0$.

if the tension in BP is not > 0 then this means the string will become slack which implies P will <u>not</u> move in circular motion.

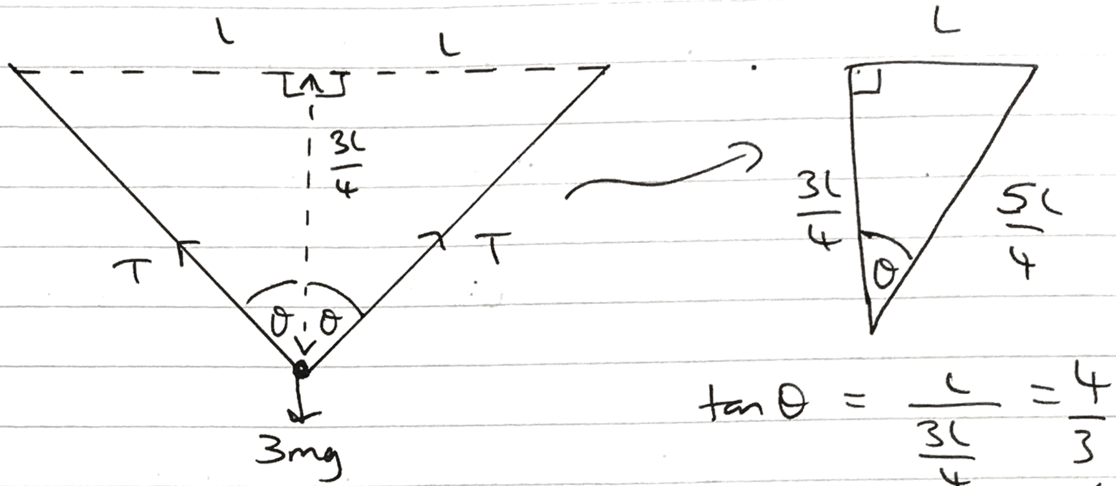
$$\Rightarrow \quad l\omega^2 - g\sqrt{2} > 0$$

$$\Rightarrow \quad \omega^2 > \frac{g\sqrt{2}}{l}$$

$\square$

$\sqrt{l^2+\left(\frac{3l}{4}\right)^2} = \frac{5l}{4}$

(Q6a)



$\tan\theta = \dfrac{l}{\frac{3l}{4}} = \dfrac{4}{3}$

$R\updownarrow : T\cos\theta + T\cos\theta = 3mg$ $\quad\therefore \cos\theta = \frac{3}{5}$

$2T\cos\theta = 3mg$

$T = \dfrac{3mg}{2\cos\theta} = \dfrac{3mg}{\frac{6}{5}} = \dfrac{5mg}{2}$

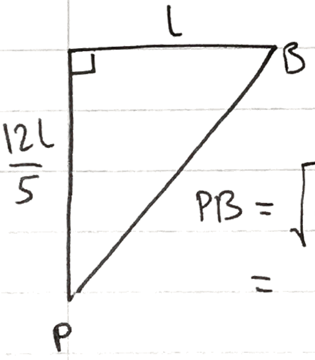
$\dfrac{5mg}{2} = \dfrac{\lambda x}{l} : \dfrac{5mg}{2} = \dfrac{kmg\left(\frac{5l}{4} - l\right)}{l}$

$\Rightarrow \dfrac{5\cancel{l}}{2} = k\left(\dfrac{\cancel{l}}{4}\right)$

$\Rightarrow k = 4 \times \dfrac{5}{2} = \boxed{10}$

b) Initially : $KE = 0$
$GPE = 0$
$EPE = \dfrac{10mg}{2l}\left(\dfrac{13l}{5} - l\right)^2 \times 2$

$= \dfrac{10mg}{2l}\left(\dfrac{128}{25}l^2\right) = \dfrac{128mgl}{5}$



$PB = \sqrt{l^2 + \left(\frac{12l}{5}\right)^2}$

$= \dfrac{13l}{5}$

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<u>at C</u> : $KE = \frac{1}{2}(3m)v^2$

$GPE = 3mg\left(\frac{12l}{5}\right)$

$EPE = 0$ (Both strings will be at natural length)

<u>C.O.E</u> : $\frac{128\cancel{m}gl}{5} = \frac{3\cancel{m}}{2}v^2 + \frac{36\cancel{m}gl}{5}$

$$\frac{92}{5}gl = \frac{3}{2}v^2$$

$$\therefore v^2 = \frac{184}{15}gl$$

hence $\boxed{v = \sqrt{\frac{184gl}{15}}}$

Q7a) at A : $KE = \frac{1}{2}mu^2$

$GPE = mg(r\cos\alpha)$

at B : $KE = \frac{1}{2}mv^2$

$GPE = mg(r\cos\theta)$.

C.O.E : $\frac{1}{2}mu^2 + mgr\cos\alpha = \frac{mv^2}{2} + mgr\cos\theta$

$\times 2$: $u^2 + 2gr\cos\alpha = v^2 + 2gr\cos\theta$.

$(l = r)$.

$u^2 + 2gl\cos\alpha - 2gl\cos\theta = v^2$

hence $v^2 = u^2 + 2gl(\cos\alpha - \cos\theta)$

$\square$

b) at the top, $v > 0$ for P to complete a vertical circle.

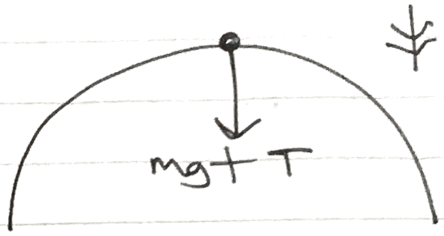
$v^2 = u^2 + 2gl\left(\frac{3}{5} - \cos 360\right) = u^2 + 2gl\left(-\frac{2}{5}\right)$

$v^2 = u^2 - \frac{4gl}{5} > 0$ //

$\therefore u^2 > \frac{4gl}{5}$

$u > 2\sqrt{\frac{gl}{5}}$

c) T is minimum at top.



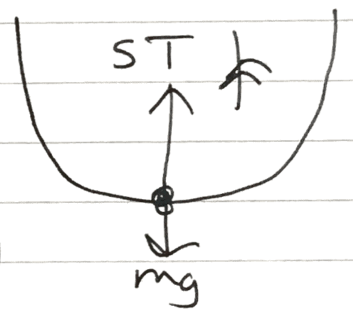
$$\text{N2L}(P) \downarrow + : \; mg + T = \frac{mv^2}{l}$$

$$T = \frac{m}{l}\left(u^2 - \frac{4gl}{5}\right) - mg$$

$$T = \frac{mu^2}{l} - \frac{4mg}{5} - mg$$

$$T = \frac{mu^2}{l} - \frac{9mg}{5} \quad \text{—— (1)}$$

T is max at bottom.



$$\text{N2L}(P) \uparrow + : \; 5T - mg = \frac{mv^2}{l}$$

$$5T - mg = \frac{m}{l}\left(u^2 + 2gl\left(\frac{3}{5}+1\right)\right)$$

$$5T - mg = \frac{mu^2}{l} + \frac{16mg}{5}$$

$$5T = \frac{mu^2}{l} + \frac{21}{5}mg \quad \text{—— (2)}$$

$$(2) = (1) \times 5 : \quad 5T = \frac{5\cancel{m}u^2}{l} - \frac{9\cancel{m}g(5)}{5} = \overbrace{\frac{\cancel{m}u^2}{l} + \frac{21\cancel{m}g}{5}}^{(2)}$$

$$\Rightarrow \quad \frac{5u^2}{l} - 9g = \frac{u^2}{l} + \frac{21g}{5}$$

$$\Rightarrow \quad \frac{4u^2}{l} = \frac{66}{5}g \quad \therefore \; u^2 = \frac{66g}{5}\left(\frac{l}{4}\right)$$

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$$\frac{66gL}{4\times5} = \frac{33gL}{10} = u^2$$