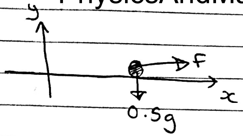
JANUARY 2010 M3

1)



$\rightarrow +$ $\quad F = ma$

$4 + \cos \pi t = 0.5a \qquad a = \frac{dv}{dt}$

$4 + \cos \pi t = 0.5 \frac{dv}{dt}$

$\int 4 + \cos \pi t . dt = 0.5 \int 1 dv$

$4t + \frac{1}{\pi} \sin \pi t \pm c = 0.5v$

When $t = 1$ $\quad v = 6 \qquad \therefore \quad c = -1$

$4 + c = 3$

$\underline{c = -1}$

$4t + \frac{1}{\pi} \sin \pi t - 1 = 0.5v$

When $t = 1.6$

$4.682\ldots = 0.5v$

$\underline{\underline{v = 9.36}}$ (3.s.f)

2)

$T = \frac{2\pi}{\omega} = 2.4 \quad \omega = \frac{2\pi}{2.4} \quad \boxed{\omega = \frac{5}{6}\pi}$

$x = a\sin\omega t$ ← use as $t=0$ at origin

~~[illegible]~~ $\frac{d}{dx}x = v$

$v = a\omega\cos\omega t$

when $t = 0.4 \quad v = 4$

$4 = a\,\frac{5}{6}\pi\cos\left(\frac{5}{6}\pi \times 0.4\right)$

$a = 3.06$ (3.s.f)

b) $v_{max} = a\omega = 3.06 \times \frac{5\pi}{6} = \underline{\underline{8}}$

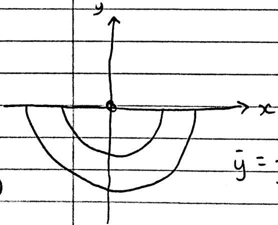
b) $a_{max} = \omega^2 a = \frac{20\pi}{3}$

3a) volume of sphere $= \frac{2}{3}\pi r^3$ centre of mass $= \frac{3}{8}r$

Volume big sphere : $\frac{2}{3}\pi r^3$ // centreom $= \frac{3}{8}r$

volume smaller : $\frac{2}{3}\pi\left(\frac{2}{3}\right)^3 = \frac{16}{81}\pi r^3$ // c.o.m $= \frac{3}{8}\times\frac{2}{3}r$

$= \frac{1}{4}r$



$\bar{y} = \frac{\Sigma my}{\Sigma m}$

	m	y
Big	$\frac{2}{3}\pi r^3$	$-\frac{3}{8}r$
small	$-\frac{16}{81}\pi r^3$	$-\frac{1}{4}r$

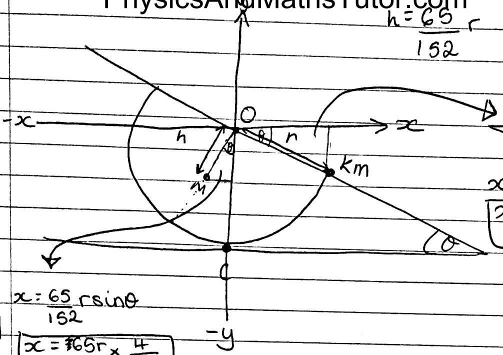
$\bar{y} = \frac{-\frac{1}{4}\pi r^4 + \frac{4}{81}\pi r^4}{\frac{38}{81}\cancel{\pi r^3}}$

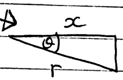
+ from O

$\bar{y} = \frac{-\frac{1}{4}r + \frac{4}{81}r}{38/\ldots} = -\frac{65}{324} \div \frac{38}{81} = \frac{-65}{152} \Rightarrow \frac{65}{152}$

$\tan\theta = \frac{4}{5}$

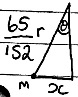
$h = \frac{65}{152} r$





$x = r\cos\theta$

$\boxed{x = r \times \frac{5}{\sqrt{41}}}$



$x = \frac{65}{152} r\sin\theta$

$\boxed{x = \frac{65r}{152} \times \frac{4}{\sqrt{41}}}$

$\bar{x} = \frac{\Sigma mx}{\Sigma m} = 0 \quad \Rightarrow \Sigma mx = 0$

$\tan\theta = \frac{4}{5}$

$\sin\theta = \frac{4}{\sqrt{41}}$

$\cos\theta = \frac{5}{\sqrt{41}}$

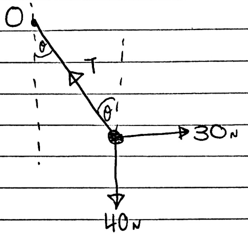
mass	x-coord
M	$-\frac{65}{152} r \times \frac{4}{\sqrt{41}}$
kM	$\frac{5r}{\sqrt{41}}$

$\Sigma mx = -\frac{\cancel{M}260\cancel{r}}{152\cancel{\sqrt{41}}} + k\cancel{M}\frac{5\cancel{r}}{\cancel{\sqrt{41}}} = 0$

$-\frac{260}{152} + 5k = 0$

$5k = \frac{260}{152}$

$\boxed{k = \frac{13}{38}}$

4) 

$l = 0.5$

$\uparrow^{+} \quad T\cos\theta = 40 \quad \ldots ①$

$\rightarrow^{+} \quad T\sin\theta = 30 \quad \ldots ②$

from ① $T = \dfrac{40}{\cos\theta}$ into ②

$\dfrac{40\sin\theta}{\cos\theta} = 30$

$\tan\theta = \dfrac{3}{4}$

$\theta = 36.9$

$T = \dfrac{40}{\cos(36.9)} = \underline{\underline{50}}$

$T = \dfrac{\lambda x}{l} \Rightarrow 50 = \dfrac{\lambda x}{0.5}$

$25 = \lambda x$

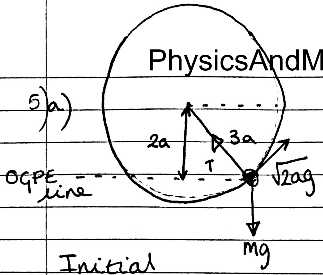
sub in

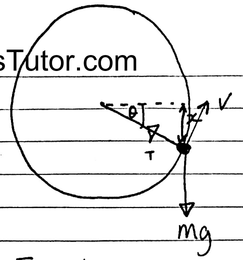
$E_{pe} = \dfrac{\lambda x^2}{2l} = \dfrac{(\lambda x)x}{2 \times 0.5}$

$10_J = \dfrac{25x}{1} \qquad x = 0.4$

length $\overrightarrow{OP} = 0.4 + 0.5 = \underline{\underline{0.9}}$

5) a)





Initial

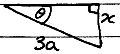
$E_k = \frac{1}{2} m 2ag$

$GPE = 0$

Final

$E_k = \frac{1}{2} m v^2$

$GPE = (2a - x) mg$



$x = 3a\sin\theta$

$\therefore \quad GPE = (2a - 3a\sin\theta) mg$

$IE = FE$

$\frac{1}{2} \cancel{m} 2ag = \frac{1}{2} \cancel{m} v^2 + (2a - 3a\sin\theta) \cancel{m}g$

$ag - g(2a - 3a\sin\theta) = \frac{1}{2} v^2$

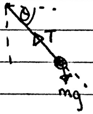
$-1ag + 3ag\sin\theta = \frac{1}{2} v^2$

$-2ag + 6ag\sin\theta = v^2$

$2ag(3\sin\theta - 1) = v^2$

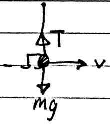
b) minimum value of T is a turning point $\therefore v = 0$

$\Rightarrow \quad 0 = 2ag(3\sin\theta - 1) \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{3}\right) = 0.3398\ldots$



$T = mg\sin\theta = mg\frac{1}{3}$

max value when $\theta = \frac{\pi}{2}$



$\uparrow^{+} \; T - mg = ma = \frac{mv^2}{r} = \frac{mv^2}{3a}$

5 b (continued) $T = \frac{mv^2}{3a} + mg$

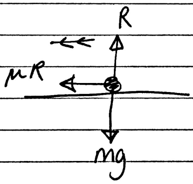
$T = \frac{m2\cancel{a}g(3\sin(\frac{\pi}{2})-1)}{3\cancel{a}} + mg$

$T = \frac{2mg \times 2}{3} + mg$

$= \frac{7mg}{3}$

$\boxed{\frac{mg}{3} \leq T \leq \frac{7mg}{3}}$

6a)



$\uparrow$ $R = mg$

$\leftarrow$ $\mu R = ma$

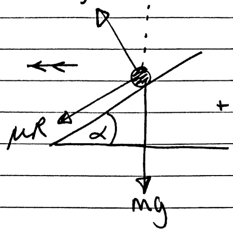
$\mu \cancel{m} g = \cancel{m} a$

$\mu g = \frac{v^2}{r}$

$\mu = \frac{28^2}{120} \div g = \boxed{\frac{2}{3}}$

b)

6b)



$\uparrow^{+}$ $R\cos\alpha - \mu R\sin\alpha = mg$ ... (1)

$+\leftarrow$ $R\sin\alpha + \mu R\cos\alpha = m\frac{v^2}{r}$ ... (2)

from (1) $R(\cos\alpha - \frac{2}{3}\sin\alpha) = mg$

$$R = \frac{mg}{(\cos\alpha - \frac{2}{3}\sin\alpha)}$$

from (2) $R(\sin\alpha + \frac{2}{3}\cos\alpha) = m\frac{1225}{120} = m\frac{245}{24}$

$$R = \frac{m\,245}{24(\sin\alpha + \frac{2}{3}\cos\alpha)}$$

equate (1) and (2) $\frac{\cancel{m}g}{(\cos\alpha - \frac{2}{3}\sin\alpha)} = \frac{245\cancel{m}}{24(\sin\alpha + \frac{2}{3}\cos\alpha)}$

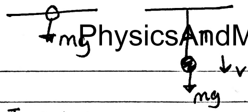
$$24g(\sin\alpha + \frac{2}{3}\cos\alpha) = 245(\cos\alpha - \frac{2}{3}\sin\alpha)$$

$$\frac{5978}{15}\sin\alpha = \frac{441}{5}\cos\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{441\times 15}{5978\times 5}$$

$$\boxed{\tan\alpha = \frac{27}{122}}$$

OGPE line



7a)

	Initial		Final
E_k:	0	E_k	$= \frac{1}{2}mv^2$
GPE:	0	GPE	$= -(a+x)mg$
E_{pe}:	0	EPE	$= \frac{\lambda x^2}{2a} = \frac{3mgx^2}{2\times 2a} = \frac{3mgx^2}{4a}$

IE = FE

$$(a+x)\cancel{m}g = \frac{1}{2}\cancel{m}v^2 + \frac{3\cancel{m}gx^2}{4a}$$

$$2g(a+x) - \frac{3gx^2}{2a} = v^2$$

b) v max when acc = 0

when $\ddot{x} = 0$ $\quad T = mg$

$$T = \frac{\lambda x}{a} = \frac{3mgx}{2a}$$

$$\frac{3\cancel{mg}x}{2a} = 1\cancel{mg} \implies x = \frac{2a}{3}$$

$$v^2 = 2g\left(a + \frac{2}{3}a\right) - \frac{3g\left(\frac{2}{3}a\right)^2}{2a}$$

$$\cancel{v^2 = \frac{98a}{3} = \frac{98a}{15}}$$

$$v^2 = 2ag + \frac{4}{3}ag - \frac{2}{3}ag$$

$$v^2 = \frac{8}{3}ag$$

$$v = \sqrt{\frac{8}{3}ag} = \frac{2}{3}\sqrt{6ag}$$

c) at rest $v=0$

$$0 = 2g(a+x) - \frac{3gx^2}{2a}$$

$$\frac{3\cancel{g}x^2}{2a} = 2\cancel{g}(a+x)$$

$$3x^2 = 4a(a+x)$$

$$3x^2 = 4a^2 + 4ax$$

$$3x^2 - 4ax - 4a^2 = 0$$

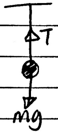
use $3x^2 - 4x - 4 = 0$

$(x-2)(3x+2)$

$\therefore \Rightarrow (x-2a)(3x+2a)$

$$x = 2a \quad \text{or} \quad x = -\frac{2}{3}a$$

$\downarrow +$ at first rest $\therefore \underline{x = 2a}$



$\downarrow$ accn direction of x increasing

$$\therefore mg - T = ma$$

$$T = \frac{\lambda x}{a} = \frac{3mg\cancel{2a}}{\cancel{2a}} = 3mg$$

$$\cancel{m}g - 3\cancel{m}g = \cancel{m}a$$

$$-2g = a$$

$$|a| = 2g$$