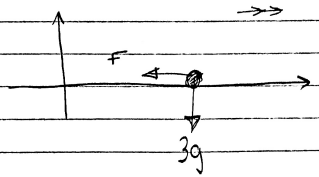
1)



$-F = ma$

$-\left(9 + \frac{15}{(t+1)^2}\right) = 3a = 3\frac{dv}{dt}$

$-\int 9 + 15(t+1)^{-2}\,dt = 3\int 1\,dv$

$-\left(9t - \frac{15}{(t+1)} + C\right) = 3v$

$-9t + \frac{15}{t+1}(+C) = 3v$

when $t = 4$ $v = 0$

$-36 + 3 + C = 0$

$C = 33$

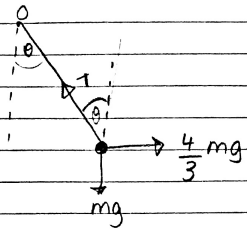
$-9t + \frac{15}{t+1} + 33 = 3v$

when $t = 0$

$\frac{15 + 33}{3} = v$

$\boxed{v = 16}$

2)



$\uparrow T\cos\theta = mg$

$\leftarrow T\sin\theta = \frac{4}{3}mg$

using pythagoras

$T^2 = (mg)^2 + (\frac{4}{3}mg)^2$

$T^2 = \frac{25}{9}m^2g^2$

$\boxed{T = \frac{5}{3}mg}$

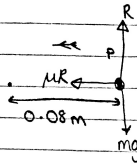
b) $Epe = \frac{\lambda x^2}{2l}$ $\quad T = \frac{5}{3}mg = \frac{\lambda x}{l}$

$\frac{5}{3}mg = \frac{3mgx}{a}$

$x = \frac{5}{9}a$

$Epe = \frac{3mg(\frac{5}{9}a)^2}{2a} = \boxed{\frac{25}{54}mga}$

3)



80 revolution p/m

$\omega = \frac{80 \times 2\pi}{60} = \frac{8\pi}{3}$

$\boxed{\omega = \frac{8\pi}{3}}$

$\uparrow^{+} R = mg$

$\leftarrow \mu R = ma$ ← least value of μ

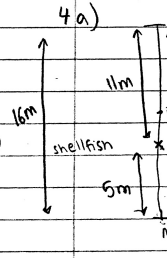
$\mu m g = m \omega^2 r$

$\mu g = \left(\frac{8}{3}\pi\right)^2 \times 0.08$

$\mu = \frac{128}{225g}\pi^2$

$\mu = 0.573$ (3 s.f)

4a)



$a = 8m$

$T = 22.5 - 10 = 12.5$

$T = \frac{2\pi}{\omega}$ $\omega = \frac{4\pi}{25}$ rad h^{-1}

run at 10 00 and 22 30

x is from centre

$\Rightarrow x = 8 - 5$

$x = 3$

$v^2 = \omega^2(a^2 - x^2)$

$v^2 = \left(\frac{4\pi}{25}\right)^2 \left(8^2 - 3^2\right)$

$v^2 = \frac{16\pi^2 \times 55}{625}$

$v = \sqrt{13.896403}$

$v = 3.7$ (2 s.f) m/h

b) $x = a\cos\omega t$ → as below centre of oscillations

↓ measured from centre = 3

$3 = 8\cos\left(\frac{4\pi}{25}t\right)$

$\cos^{-1}\left(\frac{3}{8}\right) = 1.186\ldots$ rad

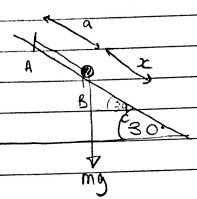
$\frac{1.186\ldots \times 25}{4\pi} = 2.36$ hours

$0.36 \times 60 = 22$ mins

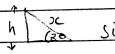
$= 2$ hours and 22 mins

$\therefore \Rightarrow 12:22$

5a)



	Initial	final
GPE:	0	$x\sin 30\, mg$
EPE:	0	$\frac{\lambda x^2}{2l} = \frac{6mgx^2}{2a}$



$\sin\theta = \frac{h}{x}$

$x\sin\theta = \Delta h$

$IE = fE$

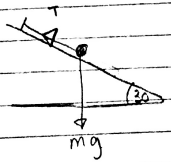
$x\sin 30\, mg = \frac{6mgx^2}{2a}$

$\frac{1}{2} = \frac{3}{a}x$

$\frac{a}{6} = x$ total length $= a\left(1 + \frac{1}{6}\right)$

$= \frac{7}{6}a$

b) V_{max} when $a = 0$



$\rightarrow$ $+ - T + mg\sin 30 = ma$

$a = 0$

$T = mg\sin 30$

$T = \frac{\lambda x}{a} = \frac{6mgx}{a} = mg\sin 30$

$\frac{6}{a}x = \frac{1}{2}$

$x = \frac{a}{12}$

	I	f
E_K	0	$\frac{1}{2}mv^2$
GPE	0	$-\frac{a}{12}\sin 30\, gm$
EPE	0	$\frac{6mg\left(\frac{a}{12}\right)^2}{2a} = 3mg \times \frac{1}{144}a = \frac{1}{24}mga$

$\frac{1}{12}\sin 30\, agm = \frac{6mg}{2a}\left(\frac{a}{12}\right)^2 + \frac{1}{2}mv^2$

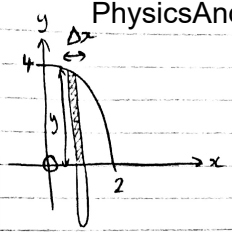
$\frac{1}{24}agm - \frac{3mg}{a}\left(\frac{a}{12}\right)^2 = \frac{1}{2}mv^2$

$\frac{1}{24}ag - \frac{1}{48}ga = \frac{1}{2}v^2$

$\frac{1ga}{24} = v^2$

$v = \frac{\sqrt{6ga}}{12}$

6)



$$\bar{x} = \frac{\Sigma mx}{\Sigma m} = \frac{\Sigma \pi y^2 x\,dx}{\Sigma \pi y^2\,dx}$$

$$= \frac{\pi \int_0^2 x y^2\,dx}{\pi \int_0^2 y^2\,dx}$$

$y = 4 - x^2$

$y^2 = (4-x^2)(4-x^2)$

$= 16 - 8x^2 + x^4$

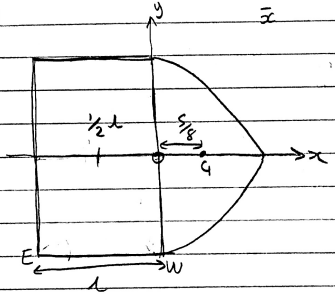
$$= \frac{\int_0^2 16x - 8x^3 + x^5\,dx}{\int_0^2 16 - 8x^2 + x^4}$$

$$= {}_0^2\left[8x^2 - 2x^4 + \frac{1}{6}x^6\right] = \frac{32}{3}$$

$${}_0^2\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right) = \frac{256}{15}$$

$$= \frac{32}{3} \div \frac{256}{15} = \boxed{\frac{5}{8}\text{ m}}$$

b)



$\bar{x}$

m	x
$\frac{256}{15}\pi$	$\frac{5}{8}$
$16\pi l$	$-\frac{1}{2}l$

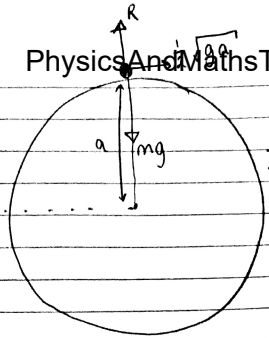
$\bar{x} = 0 \quad \therefore \frac{\Sigma mx}{\Sigma m} = 0 \quad \therefore \Sigma mx = 0$

$$\left(\frac{256}{15}\pi\right)\left(\frac{5}{8}\right) + \left(-\frac{1}{2}l\right)(16\pi l) = 0$$

$$\frac{32\pi}{3} = 8\pi l^2$$

$$\frac{4}{3} = l^2 \qquad l = \frac{2\sqrt{3}}{3} \approx 1.15$$

7 a)

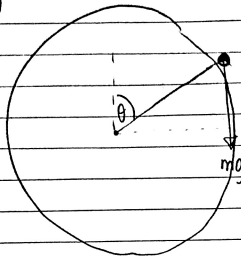


Initial

	E_k	GPE
A	$\frac{1}{2}\cdot\frac{1}{4}gam$	
	$=\frac{1}{8}agm$	amg

OGPE line

b)



Final

	E_k	GPE
B	$\frac{1}{2}mv^2$	$a\cos\theta gm$

(A) $\downarrow+$ $mg - R = ma$

$mg - R = m\frac{1}{4}g$

$R = \frac{3}{4}mg$

$R = 0$

(B) $\swarrow+$ $\cos\theta mg = \frac{mv^2}{a}$

$a\cos\theta g = v^2$

equate energies

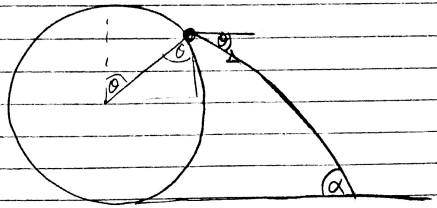
$\frac{1}{8}agm + agm = \frac{1}{2}mv^2 + a\cos\theta gm$

$\frac{9}{8}ag = \frac{1}{2}ag\cos\theta + ag\cos\theta$

$\frac{9}{8} = \frac{3}{2}\cos\theta$

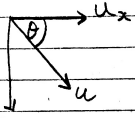
$\cos\theta = \frac{3}{4}$

b)



$u^2 = ag\cos\theta$ $\quad u = \sqrt{ag\frac{3}{4}}$

$u^2 = ag\frac{3}{4}$



$u_x = u\cos\theta = \sqrt{ag\frac{3}{4}} \times \frac{3}{4} = \frac{3\sqrt{3g}\sqrt{a}}{8}$

$u_y = u\sin\theta = \sqrt{ag\frac{3}{4}} \times \frac{\sqrt{7}}{4} = \frac{\sqrt{21g}\sqrt{a}}{8}$

↓+ $s = a + \frac{3}{4}a$

$u = u_y$

$v = v_y$

$a = 9.8/g$

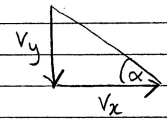
t

$v_y^2 = u_y^2 + 2g\frac{7}{4}a$

$v_y^2 = \left(\frac{\sqrt{21ag}}{8}\right)^2 + \frac{7}{2}ga$

$v_y^2 = \frac{21}{64}ag + \frac{7}{2}ag = \frac{245}{64}ag$

$V_x = u_x = \frac{3\sqrt{3ga}}{8}$



$\tan\alpha = \frac{V_y}{V_x} = \sqrt{\frac{245ag}{64}} \div \frac{3\sqrt{3ga}}{8}$

$= \frac{7\sqrt{5}}{8} \div \frac{3\sqrt{3}}{8}$

$\approx 3.0123\ldots$

$\alpha = \tan^{-1}(3.0123\ldots) = 72°$