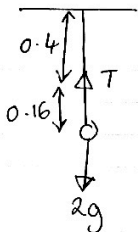


1) a)



$$\uparrow^+ \quad T - 2g = 0$$

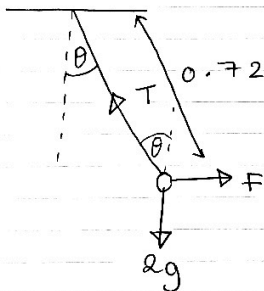
$$T = 2g$$

$$T = \frac{\lambda x}{l}$$

$$2g = \frac{\lambda \cdot 0.16}{0.4}$$

$$\lambda = 49 \text{ N}$$

b)



$$T = \frac{49 \times 0.32}{0.4}$$

$$T = 39.2$$

$$\uparrow^+ : T \cos \theta - 2g = 0$$

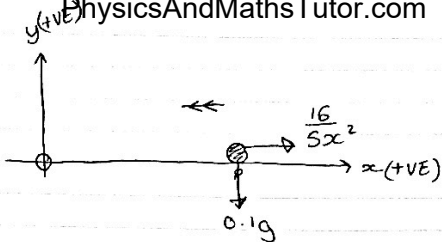
$$T \cos \theta = 2g$$

$$\# 39.2 \cos \theta = 2g$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

2)



$$+ \leftarrow F = ma$$

$$-\frac{16}{5x^2} = 0.1a$$

$$-\frac{32}{x^2} = a$$

$$a = v \frac{dv}{dx}$$

$$-\int 32x^{-2} = \int v dv$$

$$\frac{32}{x} + c = \frac{1}{2} v^2$$

$$+ \leftarrow \text{when } v=8 \quad x=-2$$

$$C + \frac{32}{-2} = \frac{1}{2} (8)^2$$

$$C = 32 + 16 = 48$$

$$\frac{32}{x} + 48 = \frac{1}{2} v^2$$

$$\text{at rest } v=0$$

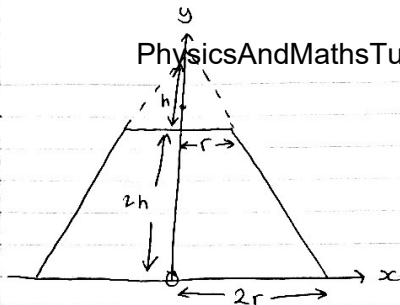
$$+ \leftarrow \text{as decelerating}$$

$$-\frac{32}{x} + 48 = 0$$

$$48 = \frac{32}{x}$$

$$\boxed{x = \frac{2}{3} \text{ m}}$$

3a



$$\text{Volume big cone} = \pi r^2 \frac{h}{3} = \pi (2r)^2 \frac{2h}{3} = \frac{8\pi r^2 h}{3}$$

$$\text{Volume small cone} = \pi r^2 \frac{h}{3} = \pi r^2 h \frac{1}{3}$$

centre mass = $\frac{1}{4}h$ from base

$$\text{big cone} = \frac{1}{4} \times 2h = \frac{1}{2}h$$

$$\text{small cone} = \frac{1}{4}h + h = \frac{5}{4}h$$

	m	y
Big	$+\frac{8}{3}\pi r^2 h$	$\frac{1}{2}h$
small	$-\frac{1}{3}\pi r^2 h$	$\frac{5}{4}h$

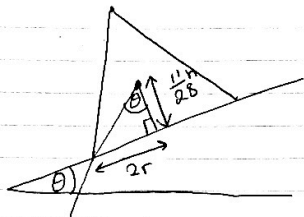
$$\bar{y} = \frac{\sum my}{\sum m}$$

$$\bar{y} = \frac{\frac{8}{3}\pi r^2 h \times \frac{1}{2}h - \frac{1}{3}\pi r^2 h \times \frac{5}{4}h}{\frac{8}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h}$$

$$= \frac{\frac{11}{12}\pi r^2 h^2}{\frac{7}{3}\pi r^2 h}$$

$$= \frac{11}{12}h \div \frac{7}{3} = \boxed{\frac{11}{28}h = \bar{y}}$$

3b)

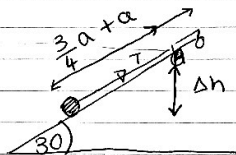
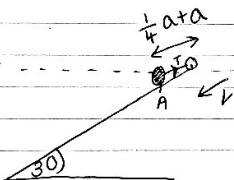


$$h = 2r$$

$$\tan \theta = \frac{2r}{\frac{11}{28}h} = \frac{h}{\frac{11}{28}h} = \frac{28}{11}$$

$$\theta = \tan^{-1}\left(\frac{28}{11}\right) = \underline{\underline{68.6^\circ}}$$

4a

 OGPE
line


Initial

$$IE_k = \frac{1}{2}mv^2$$

$$IQPE = 0$$

$$IEPE = \frac{\lambda x^2}{2l}$$

$$= \frac{2mg\left(\frac{1}{4}a\right)^2}{2a} = \frac{1}{16}mga$$

Final

$$FE_k = 0$$

$$FGPE = -mg \Delta h = -mg \frac{1}{2} a \sin 30^\circ$$

$$= -\frac{1}{4}mga$$

$$FEPE = \frac{\lambda x^2}{2l}$$

$$= \frac{2mg\left(\frac{3}{4}a\right)^2}{2a} = \frac{9}{16}mga$$

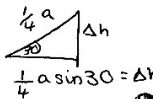
$$IE = FE$$

$$\frac{1}{2}mv^2 + \frac{1}{16}mga = \frac{9}{16}mga - \frac{1}{4}mga$$

$$\frac{1}{2}v^2 = \frac{1}{4}ga$$

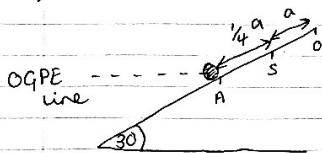
$$v^2 = \frac{1}{2}ga$$

$$v = \sqrt{\frac{ga}{2}}$$



slack $\Rightarrow T=0 \therefore x=0$

4b)



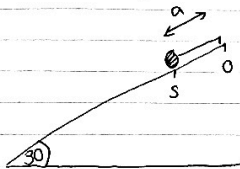
Initial

$$E_k = \frac{1}{2} m \left(\sqrt{\frac{ga}{2}} \right)^2 \left(\text{from part a} \right)$$

$$= \frac{1}{4} m g a$$

$$EPE = \frac{1}{16} m g a$$

$$GPE = 0$$



Final

$$E_k = \frac{1}{2} m u^2$$

$$EPE = 0$$

$$GPE = \frac{1}{4} a \sin 30 m g$$

$$= \frac{1}{8} m g a$$

$$IE = FE$$

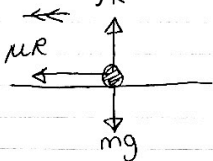
$$\frac{1}{4} m g a + \frac{1}{16} m g a = \frac{1}{2} m u^2 + \frac{1}{8} m g a$$

$$\frac{3}{16} g a = \frac{1}{2} u^2$$

$$\frac{3}{8} g a = u^2$$

$$u = \sqrt{\frac{3 g a}{8}}$$

5a)



$$+ \leftarrow F = ma$$

$$\mu R = ma$$

$$+ \uparrow R = mg$$

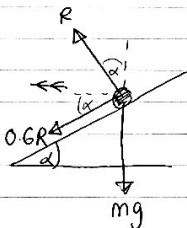
$$\mu mg = ma$$

$$\mu g = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

$$\mu = \frac{(21)^2}{75} \div g = \underline{\underline{0.6}}$$

b)



$$+ \uparrow R \cos \alpha - 0.6R \sin \alpha = mg$$

$$R \frac{4}{5} - 0.6R \frac{3}{5} = mg$$

$$R \left(\frac{4}{5} - \frac{9}{25} \right) = mg$$

$$R \frac{11}{25} = mg$$

$$R = \frac{25}{11} mg$$

c) $+ \leftarrow : F = ma$

$$0.6R \cos \alpha + R \sin \alpha = ma$$

$$0.6R \cdot \frac{4}{5} + \frac{3}{5}R = \frac{27}{25}R = ma$$

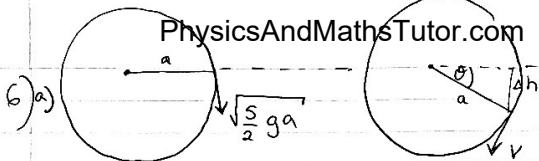
$$R = \frac{25}{11} mg \quad \therefore$$

$$\frac{27}{25} \cdot \frac{25}{11} mg = ma$$

$$\frac{27}{11} g = a = \frac{v^2}{r} = \frac{v^2}{44}$$

$$v^2 = 1058.4$$

$$v = \sqrt{1058.4} = \underline{\underline{32.5}} \text{ (3 s.f.)}$$

0 GPE
line

Initial

$$E_k = \frac{1}{2} m \left(\frac{5}{2} ga \right)$$

$$= \frac{5}{4} mga$$

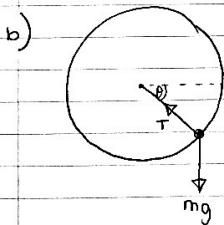
$$IE = FE$$

$$\frac{5}{4} mga = \frac{1}{2} mv^2 - a \sin \theta mg$$

$$ga \left(\frac{5}{4} + \sin \theta \right) = \frac{1}{2} v^2$$

$$2 \times \frac{ga}{4} (5 + 4 \sin \theta) = v^2$$

$$v^2 = \frac{ga}{2} (5 + 4 \sin \theta)$$



$$T - mg \sin \theta = ma = \frac{m v^2}{r}$$

$$T - mg \sin \theta = \frac{m}{a} \times \frac{ga}{2} (5 + 4 \sin \theta)$$

$$T = \frac{mg}{2} (5 + 4 \sin \theta) + mg \sin \theta$$

$$T = \frac{mg}{2} (5 + 4 \sin \theta + 2 \sin \theta)$$

$$T = \frac{mg}{2} (5 + 6 \sin \theta)$$

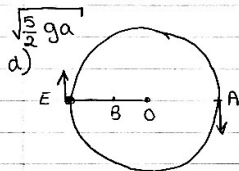
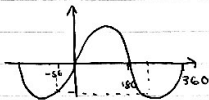
- c) when slack $\theta = a$
 slack $\Rightarrow T = 0$

$$0 = \frac{mg}{2} (5 + 6 \sin \theta) \quad \sin \theta = -\frac{5}{6}$$

$$\theta \approx -56.44$$

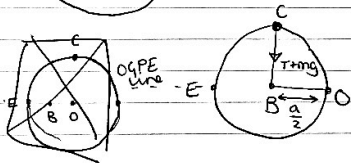
$$\theta \approx 56.44 + 180$$

$$\approx 236.4^\circ \quad (1 \text{ d.p.})$$



because conservation of E,
 speed at E = speed at A
 $= \sqrt{\frac{5}{2}ga}$

$$E_k + GPE_{at E} = E_k + GPE_{at A}$$



$$\frac{1}{2} m \frac{5}{2} ga = \frac{1}{2} mv^2 + \frac{1}{2} am$$

$$\frac{5}{4} mga - \frac{1}{2} mg = \frac{1}{2} mv^2$$

$$a \frac{3}{4} g = \frac{1}{2} v^2$$

$$v^2 = 14.7a$$

$$\downarrow : f = ma$$

$$T + mg = ma$$

$$T + mg = m \frac{v^2}{r}$$

$$T + mg = m \frac{14.7a}{a/2}$$

$$T = m(29.4 - 9.8)$$

$$T = 19.6m$$

$$T = \frac{\lambda x}{l} = 98(x+0.2)$$

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$$F = ma = m\ddot{x}$$

$$\downarrow : 2g - T = 2\ddot{x}$$

$$2g - 98(x+0.2) = 2\ddot{x}$$

$$-98x = 2\ddot{x}$$

$$-49x = \ddot{x}$$

in form

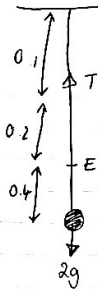
$$\ddot{x} = -\omega^2 x$$

= SHM

$$\omega = \sqrt{49} = 7$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{7} \text{ s}$$

7a)



b) $\text{max}_a = \omega^2 \times \text{max}_x = 49 \times 0.4 = 19.6 \text{ ms}^{-2}$

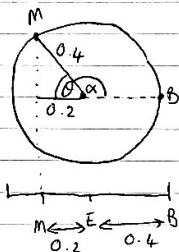
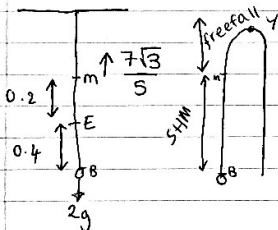
c) when slack $T=0$ $98(x+0.2) = 0$ $x = -0.2$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v^2 = 49(0.4^2 - 0.2^2)$$

$$v^2 = 5.88 \quad v = \frac{7\sqrt{3}}{5}$$

d) string slack at point (M)



from $m \rightarrow y$

$$s = \quad v = u + at$$

$$u = \frac{7\sqrt{3}}{5} \quad -\frac{7\sqrt{3}}{5} = -9.8t$$

$$v = 0 \quad t = \frac{\sqrt{3}}{7}$$

$$a = -9.8 \quad t = \frac{\sqrt{3}}{7}$$

$$t = ?$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\pi - \frac{\pi}{3} = \alpha = \frac{2\pi}{3}$$

$$\omega = \frac{\theta}{T} \quad T = \frac{\theta}{\omega} = \frac{\frac{2\pi}{3}}{\frac{7}{2\pi}}$$

$$t_{\text{from } B \rightarrow m} = \frac{2\pi}{21}$$

$$\text{total} = 2 \times t_{B \rightarrow m} = 2 \left(\frac{2\pi}{21} + \frac{\sqrt{3}}{7} \right) = 1.09 \text{ (3 s.f.)}$$