M3 - January 2007

1- a) At $x=0$, $a=0$ so max speed is reached

b) $a = \frac{1}{12}(30-x)$

$$\frac{v\,dv}{dx} = \frac{1}{12}(30-x)$$

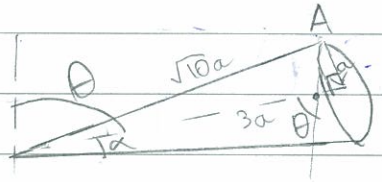
$$\int_{10}^{v} v\,dv = \frac{1}{12}\int_{30}^{x}(30-x)\,dx$$

$$\frac{1}{2}\left[v^2\right]_{10}^{v} = \frac{1}{12}\left[30x - \frac{1}{2}x^2\right]_{30}^{x}$$

$$v^2 - 100 = \frac{1}{6}\left(30x - \frac{1}{2}x^2 - 900 + 450\right)$$

$$v^2 = -\frac{1}{12}x^2 + 5x + 25$$

2-



$$\tan\theta = \frac{a}{\frac{3a}{4}} = \frac{4}{3}$$

$$\theta = 53.1° \text{ (1dp)}$$

3- a) $\bar{x} = \frac{I_{x3}}{2a} = \frac{36mg \times a^2}{2 \times a \times 3^2} = \frac{mga}{5}$

b) $\uparrow mg = R$

$F = \mu R$

$= \mu mg$

$$mE_0 = mE_A + WD$$

$$\frac{1}{2}m(2\sqrt{ag}) = \frac{mga}{5} + \frac{4}{3}a\mu mg$$

$$1 = \frac{1}{5} + \frac{4}{3}\mu$$

$$\frac{4}{5} = \frac{4}{3}\mu$$

$$\frac{3}{5} = \mu$$

4- a) $KE_A = KE_B$

$$\frac{1}{2}m3ag = \frac{1}{2}mv^2 + mg(a + a\cos\theta)$$

$$3ag = v^2 + 2ga + 2ga\cos\theta$$

$$v^2 = ag - 2ag\cos\theta$$

b) $[F = ma]$

$$T + mg\cos\theta = \frac{mv^2}{r}$$

$$T = \frac{m}{a}(ag - 2ag\cos\theta) - mg\cos\theta$$

$$= mg - 2mg\cos\theta - mg\cos\theta$$

$$= mg - 3mg\cos\theta = (1 - 3\cos\theta)mg$$

c) $0 = (1 - 3\cos\theta)mg$

$0 = 1 - 3\cos\theta$

$\cos\theta = \frac{1}{3}$

Height $= a + a\cos\theta$

$= a + \frac{1}{3}a = \frac{4}{3}a$

d) $v^2 = ag - 2ag \times \frac{1}{3} = ag - \frac{2}{3}ag = \frac{1}{3}ag$

$$v = \sqrt{\frac{1}{3}ag}$$

$$v_x = \sqrt{\frac{1}{3}ag}\cos\theta = \frac{1}{3}\sqrt{\frac{1}{3}ag}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + mgh$$

$$\frac{1}{6}ag = \frac{1}{2} \cdot \frac{1}{9} \cdot \frac{1}{3}ag + gh$$

$$h = \frac{a}{6} - \frac{a}{54} = \frac{4a}{27}$$

5 a) $\uparrow mg = T\cos\theta$ $\leftarrow$ [$F = ma$]

$T = \frac{mg}{\cos\theta}$

$T + T\sin\theta = mw^2 r$

$\frac{mg}{\cos\theta} + \frac{mg\sin\theta}{\cos\theta} = mw^2 h\sin\theta$

$\tan\theta = \frac{r}{h}$

$r = h\tan\theta$

$g + g\sin\theta = w^2 h\sin\theta$

$g(1+\sin\theta) = w^2 h\sin\theta$

$w^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$

b) $\frac{hw^2}{g} = \frac{1+\sin\theta}{\sin\theta}$

$\frac{1+\sin\theta}{\sin\theta} = \frac{1}{\sin\theta} + 1$

$\therefore \frac{1}{\sin\theta} + 1 > 2$

$\sin\theta < 1$

$1 < \frac{1}{\sin\theta}$

$\frac{hw^2}{g} > 2$

$\frac{2g}{h} < w^2$

$w^2 > \sqrt{\frac{2g}{h}}$

c) $w = \sqrt{\frac{3g}{h}}$

$w^2 = \frac{3g}{h}$

$\frac{3g}{h} = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$

$3\sin\theta = 1 + \sin\theta$

$\sin\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta = 1$

$1 - \frac{1}{4} = \cos^2\theta$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$T = \frac{mg}{\cos\theta}$

$= \frac{2mg}{\sqrt{3}}$

6 a) $V = \int_1^2 \pi y^2\,dx = \int_1^2 \frac{\pi}{4} x^{-4}\,dx$

$= \frac{\pi}{4}\left[-\frac{1}{3}x^{-3}\right]_1^2 = \frac{\pi}{4}\left(-\frac{1}{24} + \frac{1}{3}\right)$

$= \frac{7\pi}{96}$

$V(1+d) = \int_1^{2+d} \frac{\pi}{4} x^{-4}\,dx = \frac{\pi}{4}\left[-\frac{1}{3}x^{-3}\right]_1^{2+d}$

$\frac{7\pi}{96} + \frac{7\pi}{96}d = \frac{\pi}{4}\left(-\frac{1}{8} + \frac{1}{2}\right) = \frac{3\pi}{32}$

$d = \left(\frac{3}{32} - \frac{7}{96}\right) \div \frac{7}{96} = \frac{2}{7}$ m

b) $\frac{7\pi}{96} \times \frac{5}{7} + \left(1 + \frac{1}{2} \times \frac{3}{8}\right)\frac{2}{3}\pi\left(\frac{1}{2}\right)^3 = \left[\frac{7\pi}{96} + \frac{2}{3}\pi\left(\frac{1}{2}\right)^3\right]\bar{d}$

$\frac{5\pi}{96} + \frac{19}{16} \cdot \frac{2}{3} \cdot \frac{1}{8}\pi = \left(\frac{7\pi}{96} + \frac{1}{12}\pi\right)\bar{d}$

$\frac{5}{96} + \frac{19}{192} = \frac{5}{32}\bar{d}$

$\bar{d} = \frac{32 \times 29}{192 \times 5} = \frac{29}{30}$ m

7 a) $\uparrow mg = T$

$T = 0.25g$

$= 2.45$

$T = \frac{\lambda x}{a}$

$2.45 = \frac{0.05\lambda}{0.8}$

$\lambda = \frac{0.8 \times 2.45}{0.05} = 39.2$

b) $T = \frac{\lambda e}{a} = \frac{39.2 \times (0.05 + x)}{0.8} = 2.45 + 49x$

$[F = ma]$

$mg - T = m\ddot{x}$

$0.25 \times 9.8 - 2.45 - 49x = 0.25\ddot{x}$

$2.45 - 2.45 - 49x = 0.25\ddot{x}$

$\ddot{x} = -196x$

$\therefore$ SHM with $\omega^2 = 196$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$

c) $v^2 = \omega^2(a^2 - x^2)$

$v^2 = 196(0.1^2 - 0.05^2) = 1.47$

$v = 1.21$ ms^{-1} (3sf)

d) $x = 0.1\cos 14t$

$-0.05 = 0.1\cos 14t$

$\cos 14t = -0.5$ [$v = u + at$]

$t_1 = \frac{\pi}{21}$

Total time:

$\frac{\pi}{21} + 0.$

$= 0.273$s