M3 - January 2004

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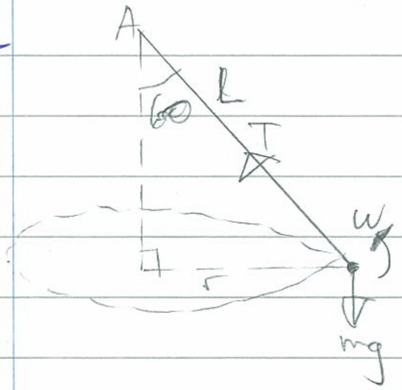


a) $\uparrow mg = T\cos 60$

$2mg = T$

b) $\leftarrow [F = ma]$

$T\sin 60 = m w^2 r$

$2mg \times \frac{\sqrt{3}}{2} = m w^2 L \frac{\sqrt{3}}{2}$

$2g = w^2 L$

$w^2 = \frac{2g}{L}$

$w = \sqrt{\frac{2g}{L}}$

c) $T = \frac{\lambda x}{a}$

$2mg = \frac{\lambda (L - \frac{3}{5}L)}{\frac{3}{5}L}$

$2mg = \left(\frac{2}{5} \times \frac{5}{3}\right)\lambda$

$\lambda = 3mg$

2 - a)

$a = -4e^{-2t}$

$\frac{dv}{dt} = -4e^{-2t}$

$\int_1^v dv = -4\int_0^t e^{-2t}\,dt$

$[v]_1^v = \frac{-4}{-2}[e^{-2t}]_0^t$

$v - 1 = 2e^{-2t} - 2$

$v = 2e^{-2t} - 1$

b) $v = 2e^{-2t} - 1$

$0 = 2e^{-2t} - 1$

$\frac{1}{2} = e^{-2t}$

$-2t = \ln \frac{1}{2}$

$t = \frac{1}{2}\ln 2$

$\frac{dx}{dt} = 2e^{-2t} - 1$

$\int_0^x dx = \int_0^{\ln\sqrt{2}} (2e^{-2t} - 1)\,dt$

$[x]_0^x = [-e^{-2t} - t]_0^{\ln\sqrt{2}}$

$x = -\frac{1}{2} - \ln\sqrt{2} + 1$

$= \left(\frac{1}{2} - \ln\sqrt{2}\right)m$

3. a) $F ~~\propto~~ \frac{1}{x^2}$ $\qquad F = \frac{k}{x^2}$

At the surface;

$\boxed{F = ma}$

$\frac{k}{R^2} = mg$

$k = mgR^2$

$\therefore F = \frac{mgR^2}{x^2}$

b) $\frac{1}{2} \times m \times u^2 = \frac{1}{2} \times m \times v^2 + \int_R^{2R} F\,dx$

$\frac{3}{4} mgR = \frac{1}{2} mv^2 + \int_R^{2R} \frac{mgR^2}{x^2}\,dx$

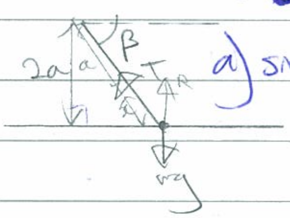
$3\cancel{m}gR = 2\cancel{m}v^2 + 4\cancel{m}gR^2 \int_R^{2R} \frac{1}{x^2}\,dx$

$3gR = 2v^2 + 4gR^2 \left[-x^{-1}\right]_R^{2R}$

$3gR = 2v^2 - \frac{4gR^2}{2R} + \frac{4gR^2}{R}$

$v^2 = \frac{1}{2}\left(3gR + \frac{4}{3}gR - 4gR\right) = \frac{1}{3}gR \times \frac{1}{2}$

$v = \sqrt{\frac{1}{6}gR}$

4. 

a) $\sin\beta = \frac{3}{5} = \frac{2a}{a+x}$

$10a = 3a + 3x$

$7a = 3x$

$x = \frac{7a}{3}$

$E = \frac{\lambda x^2}{2a}$

$= \frac{\frac{1}{2} mg \cdot \left(\frac{7a}{3}\right)^2}{2a}$

$= \frac{49mga^2}{36a}$

$= \frac{49}{36} mga$

b) $\frac{49}{36}mga = \frac{1}{2}mv^2 + \frac{mg \cdot a^2}{4a}$

$\frac{49}{18}ga - \frac{1}{2}ga = v^2$

$v = \sqrt{\frac{20}{9}ga} = \frac{2}{3}\sqrt{5ga}$

c) $T\sin\theta + R = mg$ ①

$\sin\theta = \frac{2a}{a+x}$

$T = \frac{\lambda x}{a} = \frac{mgx}{2a}$ ②

$a\sin\theta + x\sin\theta = 2a$

$x = \frac{2a - a\sin\theta}{\sin\theta}$

$= \frac{a(2-\sin\theta)}{\sin\theta}$ ③

③ in ②:

$T = \frac{mg}{2a} \cdot \frac{a(2-\sin\theta)}{\sin\theta}$

$= \frac{2mg - mg\sin\theta}{2\sin\theta}$ ④

④ in ①:

$\frac{2mg - mg\sin\theta}{2\sin\theta} \cdot \sin\theta + R = mg$

$mg(2-\sin\theta) + 2R = 2mg$

$2R = 2mg - mg(2-\sin\theta)$

$= mg(2-2+\sin\theta)$

$= mg\sin\theta$

$\sin\theta = \frac{2R}{mg}$

P remains in contact if $R \geqslant 0$

$\theta \leqslant \pi$

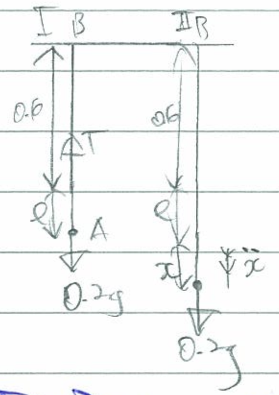
$\sin\theta \geqslant 0$

$\frac{2R}{mg} \geqslant 0$

$2R \geqslant 0$

$R \geqslant 0$

$\therefore$ Particle stays in contact

5\. 

a) $T = 0.2g$

$T = \frac{\lambda x}{a}$

$0.2g = \frac{48e}{0.6}$

$e = 0.0245$

$[F = ma]$

$0.2g - \frac{48(0.0245 + x)}{0.6} = 0.2\ddot{x}$

$0.2g - 1.96 - 80x = 0.2\ddot{x}$

$\ddot{x} = -400x$

$\therefore$ SHM with $\omega^2 = 400$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{20} = \frac{\pi}{10}$ s

b) $v_{max} = \omega r =$ ~~...~~ $20 \times 0.2755 = 5.51 \text{ ms}^{-1}$

c) $x = 0.2755 \cos 20t$

$0.1255 = 0.2755 \cos 20t$

$\cos 20t = \frac{251}{551}$

$20t = 1.098,\ (2\pi - 1.098)$

$t_1 = 0.0549$

$t_2 = 0.259$

$T_{\text{time}} = 0.259 - 0.0549$

$= 0.204$ s

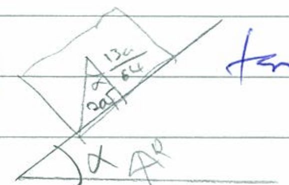
NB Proton getting eq. clearer

6\. a) $\pi(2a)^2 \cdot \frac{3}{2}a \cdot \frac{3}{4}a - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = \left[\pi(2a)^2 \cdot \frac{3}{2}a - \frac{2}{3}\pi a^3\right]\bar{d}$

$\frac{36\pi a^4}{8} - \frac{6\pi a^4}{24} = \left(\frac{12\pi a^3}{2} - \frac{2\pi a^3}{3}\right)\bar{d}$

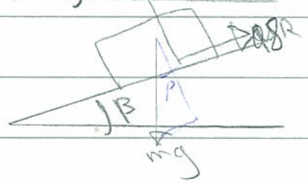
$\frac{17\pi a}{4} = \frac{16\pi}{3}\bar{d}$

$\bar{d} = \frac{17 \times 3a}{4 \times 16} = \frac{51}{64}a$

b) 

$\tan\alpha = \frac{2a}{\frac{45a}{64}} = 2a \times \frac{64}{45a} = \frac{128}{45}$

$\alpha = 70.6°$ (1dp)

c) 

$R = mg\cos\beta$

$0.8R = mg\sin\beta$

$0.8\, mg\cos\beta = mg\sin\beta$

$0.8 = \tan\beta$

$\beta = 38.7°$ (1dp)

7. a) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mg(a\sin\theta)$

$$\frac{3}{2}ga = v^2 - 2ga\sin\theta$$

$$v^2 = \frac{3}{2}ga + 2ga\sin\theta$$

b) $[F=ma]$

$$T - mg\sin\theta = \frac{m}{a}\left(\frac{3}{2}ga + 2ga\sin\theta\right)$$

$$T - mg\sin\theta = \frac{3}{2}mg + 2mg\sin\theta$$

$$T = \frac{3}{2}mg + 3mg\sin\theta = 3mg\left(\frac{1}{2} + \sin\theta\right)$$

c) When $\theta = 210$, $T = 3mg\left(\frac{1}{2} + \sin 210\right) = 3mg\left(\frac{1}{2} - \frac{1}{2}\right) = 0$

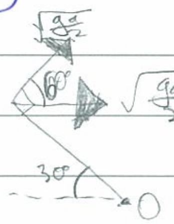
d) $\frac{1}{2}mu^2 = \frac{1}{2}\cdot\frac{3}{2}\cdot ga = \frac{3}{4}mga = KE$ initially

PE at the top: mga

$\therefore$ P cannot complete a circle because it doesn't have enough energy

e) No external work is done on the particle, so its mechanical energy is the same at the level of O

f) When $\theta = 210$, $v^2 = \frac{3}{2}ga + 2ga\sin 210 = \left(\frac{3}{2} - 1\right)ga = \frac{1}{2}ga$



At A:

$$\cos\phi = \frac{1}{2}\sqrt{\frac{ga}{2}} \times \sqrt{\frac{2}{3ga}} = \frac{1}{2\sqrt{3}}$$

$$\phi = 73.2^\circ \text{ (1dp)}$$