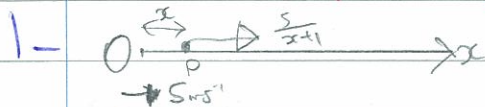


M3 - January 2002



$$[F = ma]$$

$$\frac{5}{x+1} = 0.2 \frac{v dv}{dx}$$

$$5 \int_0^x \frac{1}{x+1} dx = 0.2 \int_5^v v dx$$

$$5 [\ln|x+1|]_0^x = 0.2 \left[ \frac{1}{2} v^2 \right]_5^v$$

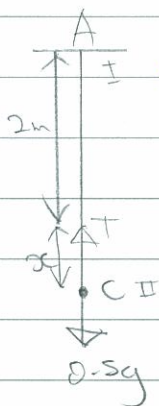
$$50 \ln|x+1| = 225 - 25$$

$$\ln|x+1| = 4$$

$$x+1 = e^4$$

$$x = e^4 - 1 = 53.0 \text{ m (3sf)}$$

2-



a) ~~Force diagram~~

~~Force diagram~~

~~Force diagram~~

~~Force diagram~~

~~Force diagram~~

$$0.5 \times 9.8 \times (2+x) = \frac{1}{2} x^2$$

$$9.8 + 4.9x = 4.9x^2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \text{ or } -1$$

$$\text{But } x > 0 \therefore x = 2$$

$$b) T = \frac{19.6 \times 2}{2}$$

$$= 19.6 \text{ N}$$

$\uparrow [F = ma]$

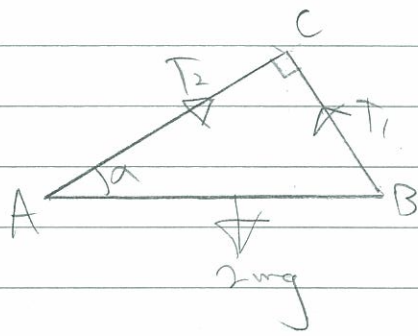
$$19.6 - 4.9 = 0.5a$$

$$14.7 = 0.5a$$

$$a = 29.4 \text{ ms}^{-2}$$

$$AC = 2 + x = 2 + 2 = 4 \text{ m}$$

3-



a)  $AC \neq AB$

b)  $2mg = T_2 \sin \alpha + T_1 \cos \alpha$  ①

$\rightarrow T_2 \cos \alpha = T_1 \sin \alpha$

$T_2 = T_1 \tan \alpha$

$T_2 = \frac{3}{4} T_1$  ②

②  $\rightarrow$  ①s

$2mg = \frac{3}{4} T_1 \sin \alpha + T_1 \cos \alpha$

$T_2 = \frac{3}{4} T_1$

$2mg = \frac{3}{4} \cdot \frac{3}{5} T_1 + \frac{4}{5} T_1$

$= \frac{3}{4} \times \frac{8mg}{5}$

$2mg = \left( \frac{9}{20} + \frac{4}{5} \right) T_1$

$= \frac{6mg}{5}$

$T_1 = \frac{2mg \times 20}{6} = \frac{8mg}{3}$

c)  $T = \frac{\lambda x}{a}$

$BC = \frac{2a \times 3}{5} = \frac{6a}{5}$

$\frac{8mg}{5} = \frac{kmgx}{a}$

$x = \frac{6a}{5} - a = \frac{a}{5}$

$k = \frac{8a}{5x}$

$k = \frac{8a}{5 \cdot \frac{a}{5}} = \frac{8a}{a} = 8$

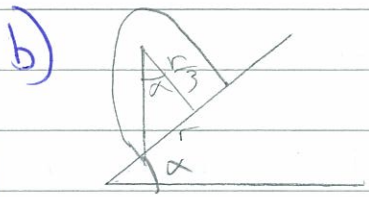
4- a)  $\int_0^r \pi y^2 x dx = \int_0^r \pi y^2 dx \cdot \bar{x}$

$\int_0^r x^2 dx = \pi r^2 \int_0^r x dx$

$\pi r \left[ \frac{1}{3} x^3 \right]_0^r = \pi r \bar{x} \left[ \frac{1}{2} x^2 \right]_0^r$

$\bar{x} = \frac{\frac{1}{3} r^3}{\frac{1}{2} r^2} = \frac{2}{3} r$

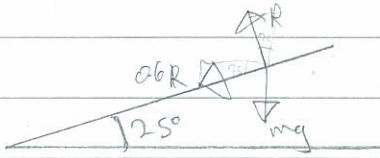




$$\tan \alpha = \frac{r}{\frac{r}{3}} = r \times \frac{3}{r} = 3$$

$$\alpha = 72^\circ \text{ (2sf)}$$

5.



$$\uparrow mg + 0.6R \sin 25 = R \cos 25$$

$$mg = R \cos 25 - 0.6R \sin 25$$

$$mg = R(\cos 25 - 0.6 \sin 25)$$

$$R = \frac{mg}{\cos 25 - 0.6 \sin 25}$$

$$\leftarrow [F = ma]$$

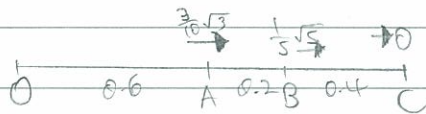
$$R \sin 25 + 0.6R \cos 25 = \frac{mv^2}{40}$$

$$\frac{mg \sin 25 + 0.6mg \cos 25}{\cos 25 - 0.6 \sin 25} = \frac{mv^2}{40}$$

$$v^2 = \frac{40g \sin 25 + 0.6 \times 40g \cos 25}{\cos 25 - 0.6 \sin 25} = 580$$

$$v = 24 \text{ ms}^{-1} \text{ (2sf)}$$

6.



a)  $v^2 = \omega^2 (a^2 - x^2)$ ,  $a = 1.2$  If SHM about O

At A

$$\left(\frac{3}{10}\sqrt{3}\right)^2 = \omega^2 (1.2^2 - 0.6^2)$$

$$\frac{27}{100} = \omega^2 (1.08)$$

$$\omega^2 = \frac{1}{4}$$

At B

$$\left(\frac{1}{5}\sqrt{5}\right)^2 = \omega^2 (1.2^2 - 0.8^2)$$

$$\frac{1}{5} = \omega^2 (0.8)$$

$$\omega^2 = \frac{1}{4}$$

b)  $v_{max} = \omega a = \frac{1}{2} \times 1.2 = 0.6 \text{ ms}^{-1}$

c)  $a = \omega^2 x = \frac{1}{4} \times 0.6 = 0.15 \text{ ms}^{-2}$

d)  $x = 1.2 \sin\left(\frac{t}{2}\right)$

$$0.6 = 1.2 \sin \frac{t}{2}$$

$$\frac{t}{2} = \frac{\pi}{6}$$

$$t = \frac{\pi}{3}$$

$$0.8 = 1.2 \sin \frac{t}{2}$$

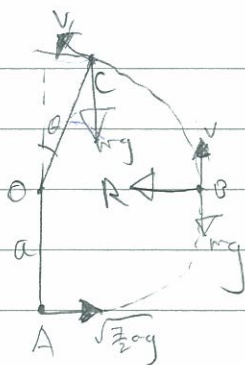
$$\frac{2}{3} = \sin \frac{t}{2}$$

$$t = 1.106$$

Time taken  $\pi$

$$1.46 - \frac{\pi}{3} = 0.425 \text{ (3sf)}$$

7



a)  $M E_A = M E_B$

$$\frac{1}{2} m \times \frac{7}{2} a g = \frac{1}{2} m v^2 + m g \cdot a$$

$$\frac{7 a g}{2} = v^2 + 2 g a$$

$$v^2 = \frac{3}{2} a g$$

$(F = ma)$

$$R = \frac{m v^2}{r} = \frac{m \cdot \frac{3}{2} a g}{2 a}$$

$$= \frac{3}{2} m g$$

b)  $M E_A = M E_C$

$$\frac{1}{2} m \cdot \frac{7}{2} a g = \frac{1}{2} m v_1^2 + m g (a + a \cos \theta)$$

$$\frac{7 a g}{2} = v_1^2 + 2 g a - 2 g a \cos \theta$$

$$v_1^2 = \frac{3}{2} a g - 2 a g \cos \theta$$

$(F = ma)$

$$m g \cos \theta = \frac{m}{r} \left( \frac{3}{2} a g - 2 a g \cos \theta \right)$$

$$g \cos \theta = \frac{3 a g}{2 a} - \frac{2 a g \cos \theta}{a}$$

$$\cos \theta = \frac{3}{2} - 2 \cos \theta$$

$$\cos \theta = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$



$$v_1^2 = \frac{3}{2} a g - 2 a g \cos \theta = \frac{3}{2} a g - 2 a g \cdot \frac{1}{2} = \frac{1}{2} a g$$

$$\text{Time taken} = \frac{a \sin \theta}{v_1 \sin \theta} = \frac{a}{v_1} = \frac{a}{\sqrt{\frac{1}{2} a g}}$$

$$= \frac{a \sqrt{2}}{\sqrt{a g}} = \frac{a \sqrt{2 a g}}{a g} = \frac{\sqrt{2 a g}}{g}$$