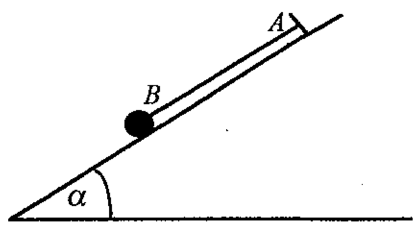
MECHANICS (C) UNIT 3 **TEST PAPER 10**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle performs simple harmonic motion along a straight line AB, about a fixed point O. The frequency of the motion is 50 oscillations per second and the amplitude of the motion is 3×10^{-2} m. Calculate the speed of the particle when it passes through O. [4]

2. A particle of mass m kg is attached to the end B of a light elastic string AB. The string has natural length l m and modulus of elasticity λ. N.
The end A is attached to a fixed point on a smooth plane inclined at an angle α to the horizontal, as shown, and the particle rests in equilibrium with the length $AB = \frac{5l}{4}$ m.

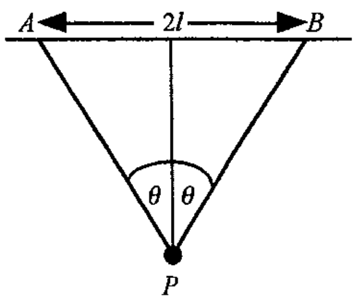


   (i) Show that $\lambda = 4\,mg \sin \alpha$. [2]

   The particle is now moved and held at rest at A with the string slack. It is then gently released so that it moves down the plane along a line of greatest slope.

   (ii) Find the greatest distance from A that the particle reaches down the plane. [5]

3. A particle P, of mass m kg, is attached to two light elastic strings, each of natural length l m and modulus of elasticity $3mg$ N. The other ends of the strings are attached to the fixed points A and B, where AB is horizontal and $AB = 2l$ m. If P rests in equilibrium vertically below the mid-point of AB, with each string making an angle θ with the vertical, show that



$$\cot \theta - \cos \theta = \tfrac{1}{6}.$$ [8]

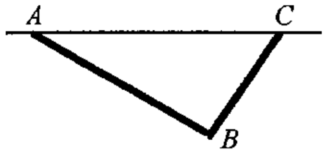
4. Suraiya, whose mass is m kg, takes a running jump into a swimming pool so that she begins to swim in a straight line with speed $0{\cdot}2$ ms^{-1}. She continues to move in the same straight line, the only force acting on her being a resistance of magnitude $mv^2 \sin\left(\frac{t}{100}\right)$ N, where v ms^{-1} is her speed at time t seconds after entering the pool and $0 \le t \le 50\pi$.

   (i) Find an expression for v in terms of t. [7]

   (ii) Calculate her greatest and least speeds during her motion. [3]

MECHANICS 3 (C) TEST PAPER 10 Page 2

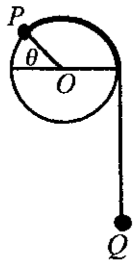
5. Two uniform rods AB and BC, of lengths $4a$ and $3a$ and weights $4mg$ and $3mg$ respectively, are smoothly jointed at B. The ends A and C are free to move on a smooth horizontal wire.



   (i) Show that, for equilibrium to be possible, the rod BC must be vertical. [5]

   (ii) Find the magnitudes of the reactions on the rods at A and C. [5]

6. The diagram shows two identical particles, each of mass m kg, connected by a thin, light inextensible string. P slides on the surface of a smooth right circular cylinder fixed with its axis, through O, horizontal. Q moves vertically. OP makes an angle θ radians with the horizontal.

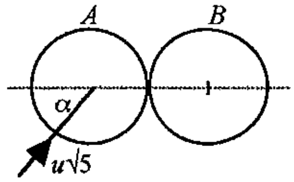


   The system is released from rest in the position where $\theta = 0$.

   (i) Show that the vertical distance moved by Q is $\frac{\theta}{\sin\theta}$ times the vertical distance moved by P. [3]

   (ii) In the position where $\theta = \frac{\pi}{6}$, prove that the reaction of the cylinder on P has magnitude $\left(1 - \frac{\pi}{6}\right)mg$ N. [7]

7. The diagram shows a smooth sphere A moving with speed $u\sqrt{5}$ striking an identical sphere B which is at rest. At the moment of impact the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{1}{2}$. The coefficient of restitution between the spheres is e.



   (i) Show that after the impact, B starts to move along the line of centres. [1]

   (ii) Show that the component of A's speed along the line of centres immediately after the impact is $(1 - e)u$. [5]

   (iii) Given further that A and B have equal kinetic energies after the impact, prove that $e = \frac{1}{4}$. [5]

MECHANICS 3 (C) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $\frac{2\pi}{\omega} = \frac{1}{50}$ $\omega = 100\pi$ $v = a\omega = 3 \times 10^{-2} \times 100\pi = 9{\cdot}42 \text{ ms}^{-1}$ M1 A1 M1 A1 **4**

2. (i) $T = mg \sin \alpha$ $\frac{\lambda}{l} \cdot \frac{l}{4} = mg \sin \alpha$ $\lambda = 4mg \sin \alpha$ M1 A1
   (ii) E.P.E. gained = grav. P.E. lost : $\frac{4mg \sin\alpha}{2l}(d - l)^2 = mg\, d \sin \alpha$ M1 A1
   $2d^2 - 5ld + 2l^2 = 0$ $(2d - l)(d - 2l) = 0$ $d = 2l$ m A1 M1 A1 **7**

3. Symmetric, so tensions in strings are equal $2T \cos \theta = mg$ B1 M1 A1
   $AP \sin \theta = l$, so $AP = \frac{l}{\sin\theta}$ $T = \frac{3mg}{l}\left(\frac{l}{\sin\theta} - l\right)$ M1 A1
   Hence $2 \times 3mg\left(\frac{1}{\sin\theta} - 1\right)\cos\theta = mg$ $6(\cot \theta - \cos \theta) = 1$, etc M1 A1 A1 **8**

4. (i) $\frac{dv}{dt} = -v^2 \sin\left(\frac{t}{100}\right)$ $\int \frac{1}{v^2}\, dv = -\int \sin\left(\frac{t}{100}\right) dt$ M1 A1
   $-\frac{1}{v} = 100 \cos\left(\frac{t}{100}\right) + c$ $t = 0, v = 0{\cdot}2 : c = -105$ A1 M1 A1
   $\frac{1}{v} = 105 - 100 \cos\left(\frac{t}{100}\right)$ $v = \frac{1}{105 - 100\cos\left(\frac{t}{100}\right)}$ M1 A1
   (ii) $v_{max} = 0{\cdot}2 \text{ ms}^{-1}$ (initial speed) $v_{min} = 0{\cdot}00952 \text{ ms}^{-1}$ $(t = 50\pi)$ M1 A1 A1 **10**

5. (i) Let Y = vertical component of reaction between rods at B,
   R, S = upward reactions from wire at A and C. B1
   If Y acts down on BC, then Y, $3mg$, S must be in same line B1
   otherwise BC rotates about mid-point. So BC is vertical. B1
   If Y acts down on AB, then AB is vertical for same reason. B1
   But this cannot happen as AB is longer. So BC is vertical. B1
   (ii) $R + S = 7mg$ M(A) for AB : $Y = 2mg$ $R = 2mg$, $S = 5mg$ B1 M1 A1 M1 A1 **10**

6. (i) P moves up $r \sin \theta$ while Q moves down by arc length $r\theta$ M1 A1
   Ratio of vertical distances moved = $\frac{r\theta}{r\sin\theta} = \frac{\theta}{\sin\theta}$ A1
   (ii) $mg \sin \theta - R = \frac{mv^2}{r}$ Energy : $mgr\theta - mgr \sin \theta = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$ M1 A1 M1 A1
   $R = mg(2 \sin \theta - \theta) = mg(1 - \frac{\pi}{6})$ when $\theta = \frac{\pi}{6}$ M1 A1 A1 **10**

7. (i) Let components after impact be p, q for A and v, w for B
   Mom. of B alone : $0 = mw$ $w = 0$, so moves along l. of c. B1
   (ii) For system, $v + p = u\sqrt{5} \cdot \frac{2}{\sqrt{5}}$ $v + p = 2u$ M1 A1
   Restitution : $v - p = -e(0 - 2u)$ Solve : $p = (1 - e)u$ M1 A1 A1
   (iii) $v = (1 + e)u$ K.E.s : $\frac{1}{2}m(1 + e)^2u^2 = \frac{1}{2}m[(1 - e)^2u^2 + u^2]$ A1 M1 A1
   $4e = 1$ $e = \frac{1}{4}$ M1 A1 **11**