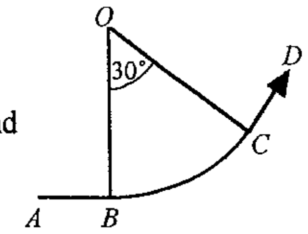
MECHANICS (C) UNIT 3 **TEST PAPER 9**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

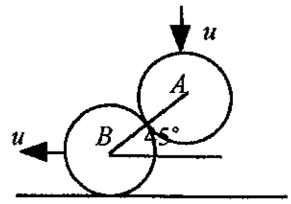
1. The diagram shows a section of a horizontal track in which AB and CD are straight and BC is an arc of a circle with centre O and radius 10 m, where angle $BOC = 30°$. A cyclist riding along the track enters the bend at B with speed 8 ms^{-1} and leaves it at C with the same speed. Modelling the cyclist and his machine as a particle P of mass 100 kg, calculate the impulse exerted on P as he rounds the bend, stating its magnitude and direction. [4]



2. A particle moving along the x-axis describes simple harmonic motion about the origin O. The period of its motion is $\frac{\pi}{2}$ seconds. When it is at a distance 1 m from O, its speed is 3 ms^{-1}. Calculate
   (i) the amplitude of its motion, [3]
   (ii) the least time that it takes to move from O to a point 0·25 m from O. [3]

3. A particle P of mass 0·5 kg moves along a straight line. When P is at a distance x m from a fixed point O on the line, the force acting on it is directed towards O and has magnitude $\frac{8}{x^2}$ N. When $x = 2$, the speed of P is 4 ms^{-1}.
   Find the speed of P when it is 0·5 m from O. [8]

4. A sphere A of mass m falls vertically so as to hit a smooth sphere B of mass km which is at rest on a horizontal table. At the moment of impact, A has speed u and the line of centres of the spheres makes an angle of 45° with the vertical. The coefficient of restitution between A and B is $\frac{1}{3}$. As a result of the impact, B starts to move horizontally with speed u.



   (i) Show that, immediately after the impact, the component of A's speed along the line of centres is $\frac{\sqrt{2}}{3}u$. [3]
   (ii) Find the component of A's new speed perpendicular to the line of centres. [2]
   (iii) Show that for the situation described to be possible, $k = \frac{1}{6}$. [4]

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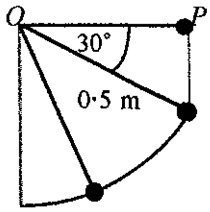
5. One end of a light inextensible string of length 0·5 m is attached to a particle P of mass 0·2 kg. The other end is attached to a fixed point O. Initially, P is held at a distance 0·5 cos 30° m horizontally from O and allowed to fall vertically.



   (i) Calculate the speed with which P first begins to move in a circle. [4]

   (ii) Show that, in the subsequent motion, when the string makes an angle of 30° with the downward vertical the tension in the string is approximately 4·6 N. [6]

6. A particle P of mass m kg is fixed to one end of a light elastic string of modulus mg N and natural length l m. The other end of the string is attached to a fixed point O on a rough horizontal table. Initially P is at rest in limiting equilibrium on the table at the point X where $OX = \frac{5l}{4}$ m.

   (i) Find the coefficient of friction between P and the table. [2]

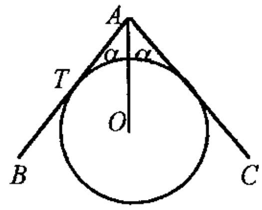
   P is now given a small displacement x m horizontally along OX, away from O. While P is in motion, the frictional resistance remains constant at its limiting value.

   (ii) Show that as long as the string remains taut, P performs simple harmonic motion with X as the centre. [4]

   If P is held at the point where the extension in the string is l m and then released,

   (iii) show that the string becomes slack after a time $\left(\frac{\pi}{2} + \arcsin\left(\frac{1}{3}\right)\right)\sqrt{\frac{l}{g}}$ s. [5]

7. The diagram shows two uniform rods AB and BC, each of length $2a$ and weight mg, smoothly jointed at A. They are placed symmetrically on a smooth fixed sphere of radius $\frac{a}{2}$ so that A is vertically above the centre O of the sphere. T (**not** the mid-point of AB) is the point of contact of AB with the sphere. Each rod makes an angle α with the vertical.



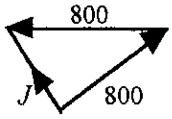
   (i) Draw a diagram showing the rod AB alone and all the forces acting on it, stating the nature of each of these forces. [3]

   (ii) Using the information from (i), show that the reaction R of the sphere on AB at T has magnitude $\frac{2mg\sin^2\alpha}{\cos\alpha}$. [3]

   (iii) By considering both rods together, show that R has magnitude $\frac{mg}{\sin\alpha}$ and deduce a trigonometric equation satisfied by α. Verify that $\alpha = 45°$ is a solution of this equation. [4]

   (iv) When $\alpha = 45°$, write down the magnitude of the reaction on AB at A. [2]

MECHANICS 3 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. Momentum Δ : 

$J = 2(800\cos 75°) = 414$ Ns — M1 A1 A1
Direction : 45° to OA — B1 — **4**

2. (i) $T = \frac{2\pi}{n}$ $\quad \frac{\pi}{2} = \frac{2\pi}{n}$ $\quad n = 4$ — B1
$v^2 = n^2(a^2 - x^2)$ $\quad 9 = 16(a^2 - 1)$ $\quad a = 1{\cdot}25$ m — M1 A1
(ii) $x = a\sin nt$ $\quad 0{\cdot}25 = 1{\cdot}25\sin 4t$ $\quad \sin 4t = 0{\cdot}2$ — M1 A1
Least t when $4t = \arcsin(0{\cdot}2)$ $\quad t = 0{\cdot}05$ s — A1 — **6**

3. $F = ma$: $0{\cdot}5v\frac{dv}{dx} = -\frac{8}{x^2}$ $\quad \int v\,dv = -\int \frac{16}{x^2}\,dx$ — B1 M1
$\frac{v^2}{2} = \frac{16}{x} + c$ $\quad x = 2, v = 4$: $8 = 8 + c$ $\quad c = 0$ $\quad v^2 = \frac{32}{x}$ — A1 M1 A1 A1
When $x = 0{\cdot}5$, $v^2 = 64$ $\quad |v| = 8$ ms^{-1} — M1 A1 — **8**

4. (i) Restitution : $-v + \frac{u}{\sqrt{2}} = e\frac{u}{\sqrt{2}}$ $\quad v = \frac{u}{\sqrt{2}}\left(1 - \frac{1}{3}\right) = \frac{\sqrt{2}}{3}u$ — M1 A1 A1
(ii) For A alone, mom. conserved ⊥ to l. of c., so component $w = \frac{u}{\sqrt{2}}$ — M1 A1
(iii) Cons. of horizontal mom. for system : $kmu + m\frac{v}{\sqrt{2}} - m\frac{w}{\sqrt{2}} = 0$ — M1 A1
$kmu = m\frac{u}{2} - \frac{1}{3}mu$ $\quad k = \frac{1}{6}$ — M1 A1 — **9**

5. (i) Vert. speed just before string becomes taut $= \sqrt{(2g \times 0{\cdot}5\sin 30°)}$ — M1 A1
$= \sqrt{(g/2)}$ Tangential component $u = \sqrt{(g/2)}\cos 30° = 1{\cdot}92$ ms^{-1} — M1 A1
(ii) Energy : $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg \times 0{\cdot}5(\cos 30° - \cos 60°)$ — M1 A1
$v^2 = 1{\cdot}92^2 + 9{\cdot}8 \times 0{\cdot}366 = 7{\cdot}27$ $\quad v = 2{\cdot}70$ ms^{-1} — A1
Then $T - mg\cos 30° = \frac{mv^2}{0{\cdot}5}$ $\quad T = 4{\cdot}61$ N — M1 A1 A1 — **10**

6. (i) Equilibrium. : $T = F$ $\quad \frac{\lambda}{l}.\frac{l}{4} = \mu mg$ $\quad \mu = \frac{1}{4}$ — M1 A1
(ii) At dist. x, $T - \mu mg = -m\ddot{x}$ $\quad \frac{mg}{l}\left(\frac{l}{4} + x\right) - \frac{1}{4}mg = -m\ddot{x}$ — M1 A1 A1
$\ddot{x} = -\frac{g}{l}x$ $\quad$ S.H.M. with $\omega^2 = \frac{g}{l}$ — A1
(iii) Amplitude $= \frac{3l}{4}$ $\quad x = \frac{3l}{4}\cos \omega t$ $\quad x = -\frac{l}{4}$: $t = \frac{1}{\omega}\arccos\left(-\frac{1}{3}\right)$ — M1 A1 M1
$= \sqrt{\frac{l}{g}}\left(\frac{\pi}{2} + \arcsin\left(\frac{1}{3}\right)\right)$ s — M1 A1 — **11**

7. (i) Diagram showing : weight of rod mg, reaction R from sphere at T, — B1 B1
X and Y : horiz. and vert. components of reaction on AB at A — B1
(or could give a single resultant reaction force at A)
(ii) M(A) for AB : $R\frac{a}{2\tan\alpha} = mg\,a\sin\alpha$ $\quad R = \frac{2mg\sin^2\alpha}{\cos\alpha}$ — M1 A1 A1
(iii) For system, resolve vert. : $2R\sin\alpha = 2mg$ $\quad R = \frac{mg}{\sin\alpha}$ — M1 A1
Equating expressions for R gives equation $2\sin^3\alpha = \cos\alpha$ — B1
When $\alpha = 45°$, LHS $= \frac{2}{(\sqrt{2})^3} = \frac{1}{\sqrt{2}}$ = RHS, so 45° is a solution — B1
(iv) By symmetry $Y = 0$, so reaction $= X = R\cos\alpha = mg$ — M1 A1 — **12**