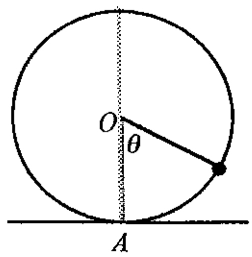
MECHANICS (C) UNIT 3 **TEST PAPER 8**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A light elastic string of natural length l m and modulus of elasticity mg N, fixed at its upper end, supports a mass m kg at its lower end. The mass is at rest with the string extended to a length $2l$ m. Find the greatest vertical speed which the mass can be given so that the string does not become slack in the subsequent motion. [4]

2. A particle of mass m kg, at rest on a smooth horizontal table, is given a horizontal impulse which causes it to start moving with speed 3 ms^{-1}. A second impulse of the same magnitude is then delivered to the particle, such that there is no change in the kinetic energy of the particle. Find the angle between the directions of the two impulses. [6]

3. A particle of mass m kg is attached to one end of a light inextensible string of length l m, whose other end is fixed to a point O. The particle is made to move in a vertical circle with centre O, with **constant** angular velocity ω rad s^{-1}. At a certain instant it is in the position shown, where the string makes an angle θ radians with the downward vertical.



   (i) Find an expression, in terms of m, l and ω, for the kinetic energy of the particle at this instant. [2]

   (ii) Find an expression, in terms of m, g, l and θ, for the potential energy of the particle relative to the horizontal plane through the lowest point A. [2]

   (iii) Determine the position of the particle when the rate of increase of its total energy, with respect to time, is a maximum. [3]

4. A light elastic string, of natural length l m and modulus of elasticity $\frac{mg}{2}$ newtons, has one end fastened to a fixed point O. A particle P, of mass m kg, is attached to the other end of the string. P hangs in equilibrium at the point E, vertically below O, where $OE = 3l$ m.

   P is now pulled down a further distance $\frac{3l}{2}$ m beyond E and is released from rest.

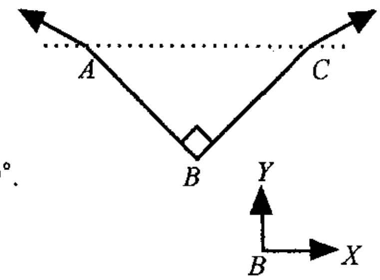
   In the subsequent motion, the string remains taut. At time t s after being released, P is at a distance x m below E.

   (i) Write down a differential equation for the motion of P and show that the motion is simple harmonic. [4]

   (ii) Write down the period of the motion. [2]

   (iii) Find the speed with which P first passes through E again. [2]

5. Two uniform rods AB and BC, of equal length but of weights $3mg$ and mg respectively, are smoothly jointed at B. They are kept at rest in a vertical plane, with A and C on the same horizontal level, by strings attached at A and C. Angle $ABC = 90°$. The horizontal and vertical components of the force on AB at B are X and Y respectively.



   (i) Show that $X + Y = \frac{3mg}{2}$. [3]

   (ii) Find the magnitude and direction of the force acting on BC at B. [7]

6. A small smooth sphere A of mass m kg is suspended from a fixed point O by a light inextensible string. A second identical sphere B falls vertically and hits A with speed u ms^{-1}. At the moment of impact, the line of centres of the spheres lies in a vertical plane and makes an angle of 45° with the vertical. Immediately after the impact, A begins to move horizontally with speed $\frac{u}{2}$ ms^{-1}. The coefficient of restitution between A and B is e.

   (i) Explain why the linear momentum of the system consisting of the two spheres is not conserved in the vertical direction. [1]

   (ii) Show that $e = \frac{1}{2}$. [11]

7. A small sphere of mass m kg is released from rest at the surface of a liquid in a right circular cylinder whose axis is vertical. The viscous resistive force on the sphere, when it is moving with speed v ms^{-1}, has magnitude v^2 N.

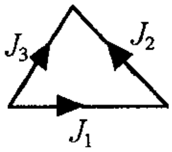
   (i) Write down a differential equation for the motion of the sphere, clearly defining any symbol(s) that you introduce. [4]

   (ii) Find, in terms of m, the distance travelled by the sphere before it attains a speed of $\sqrt{\frac{mg}{2}}$ ms^{-1}. [9]

MECHANICS 3 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1. Loss in K.E. + loss in E.P.E. ≤ loss in grav. P.E. — M1

$\frac{1}{2}mv^2 + \frac{mgl^2}{2l} \le mgl$ $\quad v^2 + gl \le 2gl$ $\quad v_{max} = \sqrt{(gl)}$ — A1 M1 A1 — 4

2. 

First impulse = J_1, second = J_2; $|J_1| = |J_2| = 3m$ — M1 A1

Final momentum is given by J_3 — B1

Since K.E. is unchanged, so is momentum : $|J_3| = 3m$ — B1

Hence Δ is equilateral and required angle = 120° — M1 A1 — 6

3. (i) K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}ml^2\omega^2$ J $\quad$ (ii) P.E. $= mgl(1-\cos\theta)$ J — M1 A1 M1 A1

(iii) Total energy $E = \frac{1}{2}ml^2\omega^2 + mgl(1-\cos\theta)$ J — M1 A1

$\frac{dE}{dt} = mgl\sin\theta\frac{d\theta}{dt} = mgl\omega\sin\theta$ $\quad$ Maximum when $\theta = \frac{\pi}{2}$ — A1 — 7

4. (i) $m\ddot{x} = mg - \frac{mg}{2l}(2l + x)$ $\quad \ddot{x} = -\frac{g}{2l}x$, so S.H.M. — M1 A1 M1 A1

(ii) $\omega^2 = \frac{g}{2l}$ $\quad$ Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{g}}$ s — M1 A1

(iii) At E, $v = a\omega = \frac{3l}{2}\sqrt{\frac{g}{2l}} = 3\sqrt{\frac{gl}{8}}$ ms^{-1} — M1 A1 — 8

5. (i) M(A) for AB: $3mg\frac{a}{\sqrt{2}} = 2X\frac{a}{\sqrt{2}} + 2Y\frac{a}{\sqrt{2}}$ $\quad X + Y = \frac{3mg}{2}$ — M1 A1 A1

(ii) M(C) for BC: $X - Y = \frac{mg}{2}$ $\quad$ Hence $X = mg$, $Y = \frac{mg}{2}$ — M1 A1 M1 A1 (both)

$R = \sqrt{1 + \frac{1}{4}}\,mg = \frac{\sqrt{5}}{2}mg$ $\quad \tan\theta = \frac{1}{2}$ $\quad \theta = 26{\cdot}6°$ to horiz. — A1 M1 A1 — 10

6. (i) There is an impulse in the string — B1

(ii) Let B have horiz. and vert. components v, w after impact

Cons. of momentum horizontally : $m\frac{u}{2} - mv = 0$ $\quad v = \frac{u}{2}$ — M1 A1

Along line of centres : $e\frac{u}{\sqrt{2}} = \frac{u}{2\sqrt{2}} - \frac{w}{\sqrt{2}} + \frac{v}{\sqrt{2}}$ $\quad v - w = eu - \frac{u}{2}$ — M1 A1 A1

Momentum for B, perp. to l. of c.: $\frac{u}{\sqrt{2}} = \frac{v}{\sqrt{2}} + \frac{w}{\sqrt{2}}$ $\quad v + w = u$ — M1 A1 A1

Combining equations, $eu - \frac{u}{2} = 0$ $\quad e = \frac{1}{2}$ — M1 A1 A1 — 12

7. (i) When distance from surface = x m, $\quad mv\frac{dv}{dx} = mg - v^2$ — B1 M1 A1 A1

(ii) $\int\frac{mv}{mg - v^2}dv = x + c$ $\quad \frac{1}{2}m\ln(mg - v^2) = x + c$ — M1 A1 M1 A1

$v = 0, x = 0 : c = -\frac{1}{2}m\ln mg$ $\quad x = \frac{1}{2}m\ln\frac{mv}{mg - v^2}$ — M1 A1 A1

When $v = \sqrt{\frac{mg}{2}}$, $\quad x = \frac{1}{2}m\ln 2$ — M1 A1 — 13

