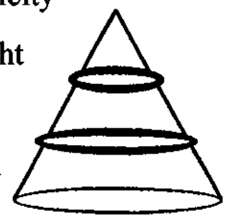
MECHANICS (C) UNIT 3 — TEST PAPER 7

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A heavy elastic ring of radius r m, natural length $2\pi r$ m and modulus of elasticity λ N has mass m kg. It is placed, with its plane horizontal, over a smooth right circular cone fixed with its axis vertical, as shown. The ring is then just taut. It is released and falls a vertical distance r m, so that in its stretched position it has radius $2r$ m, before coming to instantaneous rest. Show that $\lambda = \frac{mg}{\pi}$. [4]

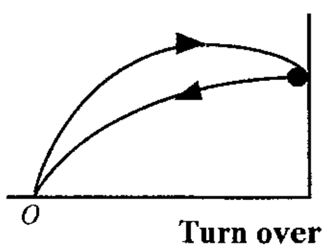


2. A racing car of mass 1000 kg, travelling on a horizontal track at 50 ms^{-1}, strikes a vertical crash barrier. It rebounds at 30 ms^{-1} at an angle of 120° to its original direction of motion.
Calculate the magnitude of the impulse given by the barrier to the car. [4]
Find the angle this impulse makes with the final direction of the car's motion. [2]

3. Two uniform rods AB and BC, each of length $2a$ and weight mg, are smoothly jointed at B. They rest in equilibrium in a vertical plane with the ends A and C on a smooth horizontal surface. The mid-points of the rods are joined by an elastic string which has modulus of elasticity λ. Each rod then makes an angle θ with the **vertical**, where $\tan\theta = \frac{3}{4}$.
   (i) State, with a reason, the magnitude of the reaction on AB at A. [2]
   (ii) Find, in terms of m and g, the tension in the string. [4]
   (iii) Given that the natural length of the string is $\frac{3a}{4}$, show that $\lambda = \frac{5mg}{4}$. [2]

4. A particle P moves in a straight line with simple harmonic motion about the point O on the line. The period of the motion is 2π seconds. When P is at the point A, at a distance of 1 m from O, its speed is 2 ms^{-1}.
   (i) Calculate the amplitude of the motion. [4]
   At a certain instant P is passing through A and moving away from O. t_0 seconds later P is again at A, but moving towards O.
   (ii) Show that $t_0 = \pi - 2\sin^{-1}\frac{1}{\sqrt{5}}$. [4]

5. A ball is projected with speed 20 ms^{-1} at an angle of elevation 15° above the horizontal. It hits a smooth vertical wall at a distance 10 m away horizontally, rebounds and returns to the point of projection O. The coefficient of restitution between the ball and the wall is e.



Turn over ...

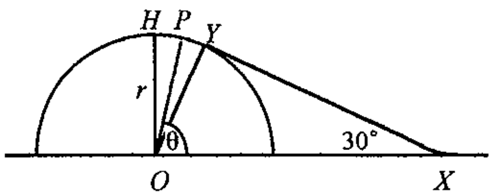
5. continued ...

   (i) State, with explanation, the values of the horizontal and vertical components of the ball's velocity when it returns to O. [4]

   (ii) By considering the horizontal and vertical motion of the ball separately, find two different expressions for the total time of flight. [6]

   (iii) Hence find the value of e, to 2 decimal places. [1]

6. The diagram shows the vertical cross-section of a smooth skating track. The ramp XY is inclined at $30°$ to the horizontal and is tangential at Y to the curved section YH of the track, which is a circular arc with centre O and radius r m in a fixed vertical plane. H is the highest point on the track.



Nevin, whose mass is m kg, skates to the ramp and passes through X with speed u ms^{-1}, after which she slides up the track without accelerating.

   (i) If she reaches H, find the minimum value of u in terms of g and r. [2]

   Given that Nevin passes through the point P, where angle $XOP = \theta$, with speed v ms^{-1},

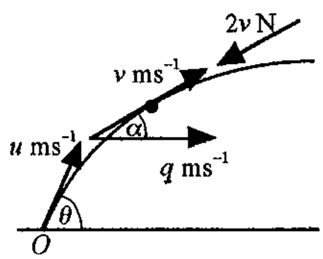
   (ii) show that the force R N exerted by the track on Nevin at P is given by

$$R = mg\sin\theta - \frac{mv^2}{r}.$$ [2]

   (iii) Use energy to express v^2 in terms of u, g, r and θ. [3]

   (iv) Find the minimum magnitude of R on the arc YH, in terms of m, u, g and r. [4]

7. A particle of mass 1 kg is projected with speed u ms^{-1} at an angle θ to the horizontal. At time t s after projection its speed is v ms^{-1} at an angle α to the horizontal, and the horizontal component of its speed is q ms^{-1}. The non-gravitational resistance at this time has magnitude $2v$ N and acts directly opposite to the instantaneous direction of motion of the particle.



   (i) Write down an expression for q in terms of v and α. [1]

   (ii) Show that $q = u\cos\theta\, e^{-2t}$. [7]

   Given that $\theta = \frac{\pi}{3}$,

   (iii) calculate the value of t when q has half the value that it would have if there were no resistance to the particle's motion. [4]

MECHANICS 3 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	Loss in grav. P.E. = gain in E.P.E. : $mgr = \frac{\lambda}{4\pi r}[(4\pi r - 2\pi r)^2]$	M1 A1 A1	
	$\lambda = \frac{2\pi mgr^2}{2\pi^2 r^2} = \frac{mg}{\pi}$	A1	4
2.	From vector triangle, with interior angle 120°, new velocity v is	M1	
	given by $v^2 = 30^2 + 50^2 - (2 \times 30 \times 50 \times \cos 120°) = 4900$ $v = 70$	M1 A1	
	$J = 7 \times 10^4$ Ns $\frac{\sin 120}{70} = \frac{\sin\alpha}{50}$ $\alpha = 38{\cdot}2°$	A1 M1 A1	6
3.	(i) Split rods at B : no vertical contact force at B, so by symmetry,	M1	
	reactions are each equal to mg	A1	
	(ii) M(B) for AB : $mg.2a.\frac{3}{5} = mg.\frac{3a}{5} + T.\frac{4a}{5}$ $T = \frac{3mg}{4}$	M1 A1 A1 A1	
	(iii) $\frac{3mg}{4} = \frac{4\lambda}{3a}\left(2a.\frac{3}{5} - \frac{3a}{4}\right)$ $\lambda = \frac{9}{16}.\frac{20}{9}mg = \frac{5mg}{4}$	M1 A1	8
4.	(i) $v^2 = n^2(a^2 - x^2)$ $T = 2\pi$, so $n = 1$ $4 = a^2 - 1$ $a = \sqrt{5}$	M1 A1 M1 A1	
	(ii) $x = \sqrt{5}\sin t$ $1 = \sqrt{5}\sin t_1$ $t_1 = \sin^{-1}\frac{1}{\sqrt{5}}$	M1 A1	
	Required time $t_0 = \frac{T}{4} + \frac{T}{4} - 2t_1 = \pi - 2\sin^{-1}\frac{1}{\sqrt{5}}$	M1 A1	8
5.	(i) 'Line of centres' perp. to wall, so $v_x = e(20\cos 15°)$	B1 B1	
	Momentum conserved vertically, so $v_y = 20\sin 15°$	B1 B1	
	(ii) Horiz. : $T = \frac{10}{20\cos 15} + \frac{10}{20e\cos 15} = 0{\cdot}52 + \frac{0{\cdot}52}{e} = 0{\cdot}52\left(1 + \frac{1}{e}\right)$	M1 A1 A1	
	Vert. (as if complete parabola) : $0 = 20\sin 15°\, t - 4{\cdot}9\, t^2$	M1 A1	
	$t = 1{\cdot}056$ (iii) Hence $e = 0{\cdot}96$	A1 A1	11
6.	(i) Energy : u is min, if just reaches H, so $\frac{mu^2}{2} = mgr$ $u = \sqrt{(2gr)}$	M1 A1	
	(ii) $\frac{mv^2}{r} = mg\sin\theta - R$ $R = mg\sin\theta - \frac{mv^2}{r}$	M1 A1	
	(iii) $\frac{mu^2}{2} = \frac{mv^2}{2} + mgr\sin\theta$ $v^2 = u^2 - 2gr\sin\theta$	M1 M1 A1	
	(iv) R is min at Y: $\theta = 60°$ $R = 3mg\sin\theta - \frac{mu^2}{r} = \frac{3mg\sqrt{3}}{2} - \frac{mu^2}{r}$	B1 M1 A1 A1	11
7.	(i) $q = v\cos\alpha$	B1	
	(ii) Equation of motion horizontally : $\frac{dq}{dt} = -2v\cos\alpha$	M1 A1	
	Using (i), $\frac{dq}{dt} = -2q$	B1	
	$\int \frac{1}{q}\,dq = -2\int dt$ $\ln q = -2t + c$	M1 A1	
	$x = 0$, $q = u\cos\theta$, so $c = \ln(u\cos\theta)$ $\ln\frac{q}{u\cos\theta} = -2t$	B1 M1	
	Hence $q = u\cos\theta\, e^{-2t}$	A1	
	(iii) With no resistance, q is constant at $u\cos\theta$, $= \frac{1}{2}u$ when $t = \frac{\pi}{3}$	B1	
	$\frac{u}{4} = \frac{u}{2}e^{-2t}$ $-2t = \ln\frac{1}{2}$ $t = \frac{1}{2}\ln 2$	M1 A1	12