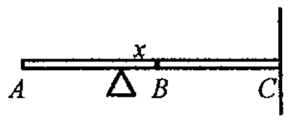
MECHANICS (C) UNIT 3 — TEST PAPER 6

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A smooth sphere A collides with an identical smooth sphere B. Before the impact A has speed u in a direction inclined at 60° to the line of centres and B is at rest. After the impact B has speed v along the line of centres, while A's speed has components q and w parallel and perpendicular to the line of centres respectively.
Given that $u = 2(q + v)$ and that the coefficient of restitution between A and B is $\frac{1}{2}$, find v in terms of u. [4]

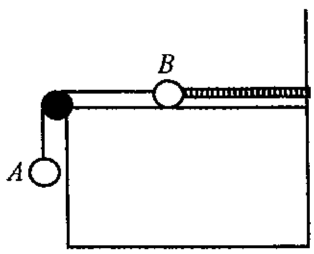
2. Two uniform beams AB, of weight $2mg$, and BC, of weight mg, are smoothly jointed at B. $AB = 5a$ and $BC = 4a$. The end C is smoothly hinged to a point on a vertical wall.



The beams are kept in equilibrium in a horizontal straight line, as shown, by a smooth support placed under AB at a distance x from B.
   (i) Find the vertical component of the force acting on BC at B, and show that the horizontal component of this force must be zero. [3]
   (ii) Find the value of x in terms of a. [3]

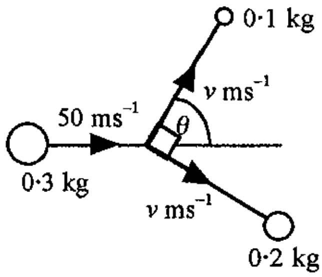
3. A small bead is threaded onto a smooth circular wire hoop, of radius r m, fixed in a vertical plane. It is then projected with speed u ms^{-1} from the lowest point of the hoop.
   (i) Find u in terms of g and r if
      (a) the bead just reaches the highest point of the hoop, [3]
      (b) the reaction on the bead is zero when it is at the highest point of the hoop. [4]
   (ii) Find, in terms of m, g, r and u, the reaction on the bead when it is at the lowest point of the hoop. [2]

4. Two particles A and B, of masses M kg and m kg respectively, are connected by a light inextensible string passing over a smooth fixed pulley. B is placed on a smooth horizontal table and A hangs freely, as shown. B is attached to a spring of natural length l m and modulus of elasticity λ N, whose other end is fixed to a vertical wall.



The system starts to move from rest when the string is taut and the spring neither extended nor compressed. A does not reach the ground, nor does B reach the pulley, during the motion.
   (i) Show that the maximum extension of the spring is $\frac{2Mgl}{\lambda}$ m. [3]
   (ii) If $M = 3$, $m = 1{\cdot}5$ and $\frac{\lambda}{l} = 35$ Nm^{-1}, find the speed of A when the extension in the spring is $0{\cdot}5$ m. [6]

5. A fireworks pod of mass 0·3 kg moving horizontally with speed 50 ms^{-1} explodes, forming two fragments of masses 0·1 kg and 0·2 kg. These fragments continue to move in the same horizontal plane, with equal speeds v ms^{-1}, in perpendicular directions as shown, with the direction of the lighter fragment inclined at an angle θ to the original direction of motion.



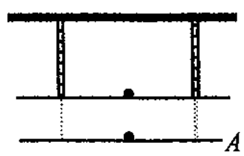
By using a vector diagram to depict conservation of linear momentum, or otherwise,

(i) show that $\tan\theta = 2$. [3]

(ii) Find the value of v. [3]

(iii) Calculate the magnitude of the impulse on the 0·1 kg mass due to the explosion. [3]

6. The figure shows a swing consisting of two identical vertical light springs attached symmetrically to a light horizontal cross-bar and supported from a strong fixed horizontal beam. When a mass of 24 kg is placed at the mid-point of the cross-bar, both springs extend by 30 cm to the position A, as shown.



Each spring has natural length l m and modulus of elasticity λ N.

(i) Show that $\lambda = 392l$. [3]

The 24 kg mass is left on the bar and the bar is then displaced downwards by a further 20 cm.

(ii) Prove that the system comprising the bar and the mass now performs simple harmonic motion with the centre of oscillation at the level A. [5]

(iii) Calculate the number of oscillations made per second in this motion. [3]

7. A particle of mass m kg moves in a straight line under the action of a variable force $F = (-18m\cos 3t)$ N, where t s is the time that has elapsed since the start of the motion. Given that, when $t = 0$, the particle has velocity 7 ms^{-1} and is at a distance of 2 m from a fixed point O on the line,

(i) find an expression in terms of t for the velocity v ms^{-1} of the particle. [5]

(ii) Write down the least and greatest values of v, stating the values of t at which these first occur. [3]

(iii) State the acceleration of the particle at the times found in (ii). [1]

(iv) Express in terms of t the displacement x m of the particle from O. [3]

MECHANICS 3 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1. Restitution : $-\frac{1}{2}(0 - u\cos 60°) = v - q$ $\quad v - q = \frac{1}{4}u$ M1 A1

Given $\frac{1}{2}u = v + q$ $\quad$ Solving simultaneously, $v = \frac{3}{8}u$ M1 A1 **4**

2. (i) Separate the rods, showing forces at joint B : X horiz. and Y vert.

M(C) for BC : $mg(2a) = Y(4a)$ $\quad Y = \frac{1}{2}mg$ (upwards) M1 A1

For AB, no horiz. force other than X, so $X = 0$ B1

(ii) Resolving vertically for AB, upward force $= 2mg + \frac{1}{2}mg = \frac{5}{2}mg$ B1

M(B) for AB : $\frac{5mg}{2}x = 2mg\frac{5a}{2}$ $\quad x = 2a$ M1 A1 **6**

3. (i) (a) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$ $\quad v = 0, h = 2r : \frac{1}{2}u^2 = 2gr$ $\quad u = 2\sqrt{(gr)}$ M1 A1 A1

(b) At top, force towards centre $= \frac{mv^2}{r}$, $= mg$ as $R = 0$ M1 A1

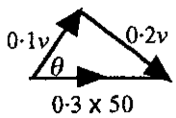
Thus $v^2 = gr$, so $mu^2 = mv^2 + 4mgr = 5mgr$ $\quad u = \sqrt{(5gr)}$ M1 A1

(ii) $R' - mg = \frac{mu^2}{r}$ $\quad R' = \frac{mu^2}{r} + mg$ M1 A1 **9**

4. (i) Loss in P.E. = gain in E.P.E. : $Mge = \frac{\lambda}{2l}e$ $\quad e = \frac{2Mgl}{\lambda}$ M1 A1 A1

(ii) Loss in P.E. of A = gain in K.E. of (A & B) + gain in E.P.E. M1 M1

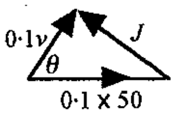
$3 \times 9{\cdot}8 \times 0{\cdot}5 = \frac{1}{2}(4{\cdot}5)v^2 + \frac{\lambda}{2l}(0{\cdot}25)$ $\quad v^2 = 4{\cdot}589$ $\quad v = 2{\cdot}14$ ms^{-1} A1 A1 M1 A1 **9**

5. (i)


From momentum triangle, B1

$\tan\theta = \frac{0{\cdot}2v}{0{\cdot}1v} = 2$ M1 A1

(ii) $0{\cdot}1v\cos\theta + 0{\cdot}2v\sin\theta = 0{\cdot}3 \times 50$ $\quad v = 67{\cdot}1$ M1 A1 A1

(iii)


$J^2 = 5^2 + 6{\cdot}71^2 - (2 \times 5 \times 6{\cdot}71 \times \cos\theta)$ M1 A1

$J = \sqrt{40} = 6{\cdot}33$ Ns A1 **9**

6. (i) $24g = 2T = 2\frac{\lambda}{l}(0{\cdot}3)$ $\quad \frac{\lambda}{l} = \frac{24 \times 9{\cdot}8}{2 \times 0{\cdot}3} = 392$ $\quad \lambda = 392l$ M1 A1 A1

(ii) At dist. x from A, $mg - 2\frac{\lambda}{l}(0{\cdot}3 + x) = m\ddot{x}$ M1 A1 A1

$\ddot{x} = -\frac{2l}{ml}x = -\frac{98}{3}x$ $\quad$ Hence S.H.M. with centre A A1 A1

(iii) $\omega^2 = \frac{98}{3} = 32{\cdot}7$ $\quad$ Freq. $= \frac{\omega}{2\pi} = \frac{\sqrt{32{\cdot}7}}{2\pi} = 0{\cdot}91$ osc. s^{-1} M1 A1 A1 **11**

7. (i) $m\frac{dv}{dt} = -18m\cos 3t$ $\quad v = -6\sin 3t + c$ $\quad v(0) = 7 : c = 7$ M1 A1 M1

$v = 7 - 6\sin 3t$ $\quad$ (ii) $v_{min} = 1$ when $t = \frac{\pi}{6}$, $v_{max} = 13$ when $t = \frac{\pi}{2}$ A1 A1; M1 A1 A1

(iii) When v is max. or min., acc = 0 B1

(iv) $\frac{dx}{dt} = 7 - 6\sin 3t$ $\quad x = 7t + 2\cos 3t + k$ $\quad x(0) = 2 : k = 0$ M1 A1

$x = 7t + 2\cos 3t$ A1 **12**