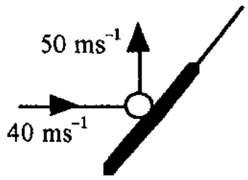
MECHANICS (C) UNIT 3 **TEST PAPER 5**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A cricket ball of mass 140 g is moving horizontally, directly towards a batsman. Its speed before it hits the bat is 40 ms^{-1}. Immediately after the impact, it leaves the bat vertically with speed 50 ms^{-1}.



Find the magnitude of the impulse of the bat on the ball and calculate its direction. Show these results clearly on an impulse-momentum triangle. [4]

2. A particle of mass m kg moves in a horizontal straight line. Its initial speed is u ms^{-1} and the only force acting on it is a variable resistance of magnitude mkv N, where v ms^{-1} is the speed of the particle after t seconds and k is a constant.
Show that $v = ue^{-kt}$. [6]

3. A particle P moves in a straight line. At time t s, its displacement x m from a fixed point O on the line is given by $x = 2\cos(0{\cdot}4t + \varepsilon)$, where ε is a constant such that $0 < \varepsilon < \pi$.
   (i) Show that P performs simple harmonic motion. [3]
   (ii) Write down the period of its motion. [1]
   Given also that $x = 0$ when $t = \frac{5\pi}{8}$,
   (iii) find the value of ε. [2]
   (iv) Find the value of x when $t = \frac{5\pi}{2}$. [2]

4. A particle P of mass 0·2 kg moves in a horizontal circle on one end of an elastic string whose other end is attached to a fixed point O. The angular velocity of P is π rad s^{-1}. The natural length of the string is 1 m and, while P is in motion, the distance $OP = 1{\cdot}15$ m.
   (i) Calculate, to 3 significant figures, the modulus of elasticity of the string. [6]
   The motion now ceases and P hangs at rest vertically below O.
   (ii) Show that the extension in the string in this position is about 13 cm. [3]

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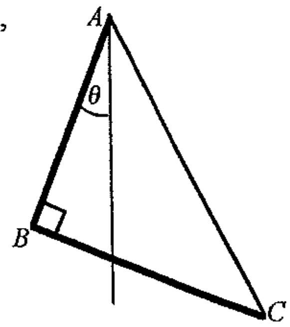
5. Two identical uniform rods AB and BC, each of length a and weight mg, are smoothly jointed at B. The end A is freely hinged to a fixed point. A light inextensible string connects A and C. The rods rest in equilibrium in a vertical plane with angle $ABC = 90°$ and AB inclined at an angle θ to the vertical, where $\tan\theta = \frac{1}{3}$.

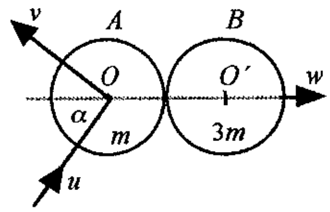


Prove that,

(i) the tension in the string has magnitude $0{\cdot}67\,mg$, correct to 2 significant figures; [4]

(ii) the reaction between the rods at B has magnitude $0{\cdot}5\,mg$. [6]

6. A smooth sphere A of mass m, at rest on a smooth horizontal table, is hit by another smooth sphere B of mass $3m$ and equal radius. Their line of centres is OO'. Before the impact, A was moving with speed u at an angle α to OO' as shown. After the impact A moves perpendicular to its original direction with speed v, and B moves along OO' with speed w. The coefficient of restitution between A and B is e.



(i) Write down an equation relating e, u, w, v and α. [1]

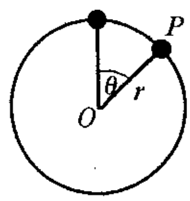
(ii) By considering the conservation of momentum of the system consisting of A and B together, derive another equation relating u, w, v and α. [2]

(iii) Hence show that $v = \dfrac{(3e-1)u\cos\alpha}{4\sin\alpha}$. [2]

(iv) Prove that $\tan^2\alpha = \dfrac{3e-1}{4}$. [3]

(v) Deduce the ranges of possible values of e and of $\tan\alpha$. [3]

7. A particle P is gently disturbed from rest at the highest point of a smooth circular hoop of radius r m fixed in a vertical plane, and moves on the surface of the hoop along the circular cross-section such that the radius OP makes an angle θ with the vertical.



(i) Show that P leaves the surface of the hoop when $\cos\theta = \frac{2}{3}$. [6]

P is now retrieved and placed inside the hoop at its lowest point. It is given a horizontal speed u ms^{-1} so that it moves on the smooth inner surface of the hoop. If it leaves the hoop at the same position as before,

(ii) find u in terms of g and r. [6]

MECHANICS 3 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1.	$\mathbf{J} = m\mathbf{v} - m\mathbf{u}$ $\lvert\mathbf{J}\rvert = 0{\cdot}14\sqrt{(40^2 + 50^2)} = 8{\cdot}96$ Ns	M1 A1	
	Direction α to horizontal is such that $\tan\alpha = \frac{5}{4}$ $\alpha = 51{\cdot}3°$	M1 A1	4
2.	$m\frac{dv}{dt} = -mkv$ $\int \frac{1}{v}\,dv = \int -k\,dt$ $\ln v = -kt + c$	M1 A1 A1	
	$\ln u = c$, so $\ln\frac{v}{u} = -kt$ $v = ue^{-kt}$	M1 A1 A1	6
3.	(i) $x = 2\cos(0{\cdot}4t + \varepsilon)$ $\dot{x} = -0{\cdot}8\sin(0{\cdot}4t + \varepsilon)$	M1	
	$\ddot{x} = -0{\cdot}16\cos(0{\cdot}4t + \varepsilon) = -0{\cdot}4^2x$ Hence SHM	A1 A1	
	(ii) Period $= \frac{2\pi}{0{\cdot}4} = 5\pi$ s (or 15·7 s)	B1	
	(iii) $0 = 2\cos(0{\cdot}4 \times \frac{5\pi}{8} + \varepsilon)$ $\cos(\frac{\pi}{4} + \varepsilon) = 0$ $\varepsilon = \frac{\pi}{4}$	M1 A1	
	(iv) When $t = \frac{5\pi}{2}$, $x = 2\cos\frac{5\pi}{4} = -1{\cdot}4$ m	M1 A1	8
4.	(i) $T = 0{\cdot}2(1{\cdot}15)\pi^2$, $T = \frac{\lambda}{1}(0{\cdot}15)$, so $\lambda = \frac{(0{\cdot}2)(1{\cdot}15)\pi^2}{0{\cdot}15} = 15{\cdot}1$ N	M1 A1 M1 A1 M1 A1	
	(ii) $T_1 = mg = 0{\cdot}2(9{\cdot}8)$ $1{\cdot}96 = \frac{\lambda}{1}x$ $x = 0{\cdot}1298$ m ≈ 13 cm	B1 M1 A1	9
5.	(i) M(B) for rod BC : $T\frac{a}{\sqrt{2}} = mg\,\frac{a}{2}\cos\theta$ $T = \frac{mg\cos\theta}{\sqrt{2}} = 0{\cdot}67\,mg$	M1 A1 A1 A1	
	(ii) For rod BC, resolving horiz. : $T\cos(45° + \theta) = X$ (vert. comp.)	M1 A1	
	$X = 0{\cdot}67\,mg\cos(45 + 18{\cdot}4)° = 0{\cdot}3\,mg$	A1	
	Resolving vertically, $T\sin(45° + \theta) + Y = mg$ $Y = 0{\cdot}4\,mg$	M1 A1	
	Reaction at $B = mg\sqrt{(0{\cdot}3^2 + 0{\cdot}4^2)} = 0{\cdot}5\,mg$	A1	10
6.	(i) Restitution : $eu\cos\alpha = w - (-v\sin\alpha)$ $eu\cos\alpha = w + v\sin\alpha$	B1	
	(ii) Mom : $mu\cos\alpha = 3mw + m(-v\sin\alpha)$ $u\cos\alpha = 3w - v\sin\alpha$	M1 A1	
	(iii) Eliminating w, $4v\sin\alpha = (3e - 1)u\cos\alpha$ $v = \frac{(3e-1)u\cos\alpha}{4\sin\alpha}$	M1 A1	
	(iv) For A alone $\perp OO'$, $v\cos\alpha = u\sin\alpha$ $\frac{(3e-1)u\cos\alpha}{4\sin\alpha} = \frac{u\sin\alpha}{\cos\alpha}$	M1 A1	
	so $\tan^2\alpha = \frac{3e-1}{4}$	A1	
	(v) $\tan^2\alpha > 0$, so $e > \frac{1}{3}$ $e \le 1$, so $-\frac{1}{\sqrt{2}} \le \tan\alpha \le \frac{1}{\sqrt{2}}$	B1 M1 A1	11
7.	(i) Energy : $\frac{1}{2}mv^2 = mgr(1 - \cos\theta)$ $mg\cos\theta - R = \frac{mv^2}{r}$	B1 M1 A1	
	Hence $R = mg\cos\theta - 2mg(1 - \cos\theta) = mg(3\cos\theta - 2)$	M1 A1	
	P leaves hoop when $R = 0$, i.e. when $\cos\theta = \frac{2}{3}$	A1	
	(ii) At this position, $\frac{1}{2}mv^2 + mgr(1 + \frac{2}{3}) = \frac{1}{2}mu^2$ $v^2 = u^2 - \frac{10}{3}gr$	M1 A1	
	$R' + \frac{2}{3}mg = \frac{m}{r}(u^2 - \frac{10}{3}gr)$	M1 A1	
	When $R' = 0$, $\frac{2mg}{3} + \frac{10mg}{3} = \frac{mu^2}{r}$ $u^2 = 4gr$ $u = 2\sqrt{(gr)}$	M1 A1	12