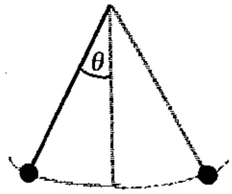
MECHANICS (C) UNIT 3 **TEST PAPER 4**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. When a simple pendulum of length l m swings through a very small amplitude, its period is t s. When it swings through a larger amplitude θ radians, the period T s is given by a better approximation



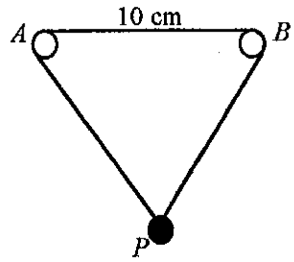
$$T \approx 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta}{2}\right).$$

(i) Show that in this case, assuming θ is small, $T \approx 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{\theta^2}{16}\right)$. [2]

(ii) Hence show that if $\frac{T-t}{t} < 0{\cdot}1$, the maximum value of θ is $0{\cdot}4$. [2]

2. A spacecraft of mass 500 kg is cruising with a speed of 3 km s^{-1}. The ground control station gives it an impulse of magnitude 10^5 Ns to cause a change of 2° in the direction of its motion. Assuming that the spacecraft's motion remains in the same plane, calculate the speed with which it moves along its new path. [7]

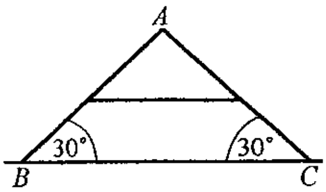
3. A thin elastic string, of modulus λ N and natural length 20 cm, passes round two small, smooth pegs A and B on the same horizontal level to form a closed loop. AB = 10 cm. The ends of the string are attached to a weight P of mass 0·7 kg. When P rests in equilibrium, APB forms an equilateral triangle.



(i) Find the value of λ. [6]

(ii) State one assumption that you have made about the weight P, explaining how you have used this assumption in your solution. [1]

4. Two uniform rods AB and BC, each of length $2a$ and weight mg, are smoothly jointed at A. Their mid-points are joined by a light inelastic string and the system rests in equilibrium with B and C on a smooth horizontal floor and each rod inclined at 30° to the floor.



(i) Show that the reaction forces at B and C are equal in magnitude, and find their magnitudes. [2]

(ii) Find the tension in the string. [3]

(iii) Show that the force acting on AB at A has magnitude $mg\sqrt{3}$, and state its direction. [2]

If, instead, the floor is rough and the rods rest in equilibrium in the same position,

(iv) state, with a reason, whether the magnitude of the tension in the string will be larger, smaller or the same as before. [2]

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5. Two identical small smooth marbles A and B, each of mass m kg, collide with each other. The table gives the components of their velocity (in ms^{-1}) before and after impact, relative to perpendicular x and y axes.

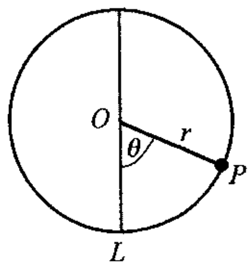
	Before impact		After impact	
	x component	*y component*	*x component*	*y component*
A	5	−8	u	v
B	8	4	p	$-q$

At the moment of impact, the line joining the centres of the marbles lies along the y-axis. The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Show that (a) $q + v = 8$, (b) $q - v = 4$. [4]

(ii) Find, in terms of m, the loss in the total kinetic energy of A and B due to the impact. [6]

6. A small bead P, of mass m kg, can slide on a smooth circular ring, with centre O and radius r m, which is fixed in a vertical plane. P is projected from the lowest point L of the ring with speed $\sqrt{(3gr)}$ ms^{-1}. When P has reached a position such that OP makes an angle θ with the downward vertical, as shown, its speed is v ms^{-1}.



(i) Show that $v^2 = gr(1 + 2\cos\theta)$. [5]

(ii) Show that the magnitude of the reaction R N of the ring on the bead is given by

$$R = mg(1 + 3\cos\theta).$$ [4]

(iii) Find the values of $\cos\theta$ when

(a) P is instantaneously at rest, (b) the reaction R is instantaneously zero. [2]

7. A particle of mass m kg moves horizontally in a medium which offers a resistance of magnitude $\dfrac{mv^2}{k+x}$ N, where x m is the distance travelled into the medium and v ms^{-1} is the speed at time t s after it enters the medium. k is a positive constant.

Given that the speed of the particle is u ms^{-1} when $t = 0$,

(i) show that $v = \dfrac{ku}{k+x}$. [6]

Given further that $x = 2$ when $t = 2$,

(ii) find k in terms of u and write down a necessary condition on u for the motion to be possible. [6]

MECHANICS 3 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1.	(i) As θ is small, $\sin\theta \approx \theta$, so $\frac{1}{4}\sin^2\frac{\theta}{2} \approx \frac{1}{4}\left(\frac{\theta}{2}\right)^2 = \frac{\theta^2}{16}$	M1 A1	
	(ii) $t = 2\pi\sqrt{\frac{l}{g}}$ so $\frac{T-t}{t} = \frac{\theta^2}{16} < 0\cdot1$ $\quad\theta < 0\cdot4$	M1 A1	4
2.	Sine rule in momentum Δ : $\frac{10^5}{\sin 2^\circ} = \frac{500\times3\times10^3}{\sin\theta}$ $\quad\theta = 31\cdot6°$	M1 A1 M1 A1 A1	
	Then $\frac{10^5}{\sin 2} = \frac{500v}{\sin(180-33\cdot6)}$ $\quad v = 3\cdot2 \times 10^3$ ms^{-1} = 3·2 km s^{-1}	M1 A1	7
3.	(i) $T = \frac{\lambda(0\cdot1)}{0\cdot2} = \frac{\lambda}{2}$ $\quad$ Resolve vertically : $2T\cos 30° = 0\cdot7g$	M1 A1 M1 A1	
	$T = 3\cdot96$ $\quad\lambda = 2T = 7\cdot92$	M1 A1	
	(ii) Assumed P is a particle, e.g. a single point at vertex of Δ	B1	7
4.	(i) Reactions equal by symmetry; $2R = mg + mg$ $\quad R = mg$	B1 B1	
	(ii) M(A) for AB : $mg(2a\cos 30°) = mg(a\cos 30°) + T(a\sin 30°)$	M1 A1	
	$mg\sqrt{3} = mg\frac{\sqrt{3}}{2} + \frac{T}{2}$ $\quad T = mg\sqrt{3}$	A1	
	(iii) For AB alone, vertical force = 0, horiz. force = $T = mg\sqrt{3}$	M1 A1	
	(iv) Friction acts in direction to help tension pulling on each rod,	M1	
	so tension will be less	A1	9
5.	(i) Restitution : $-e(12) = -q - v$ $\quad q + v = 8$	M1 A1	
	Momentum for system : $-8 + 4 = v - q$ $\quad q - v = 4$	M1 A1	
	(ii) Solve eqns : $q = 6$, $v = 2$ $\quad$ Also, must have $p = 8$, $u = 5$	M1 A1 B1 B1	
	K.E. lost $= \frac{1}{2}m[5^2 + 8^2 + 8^2 + 4^2 - (5^2 + 8^2 + 6^2 + 2^2)] = 20m$ J	M1 A1	10
6.	(i) P.E. gained = K.E. lost : $mgr(1 - \cos\theta) = \frac{1}{2}m(3gr) - \frac{1}{2}mv^2$	M1 M1 A1	
	$v^2 = 3gr - 2gr + 2gr\cos\theta = gr(1 + 2\cos\theta)$	M1 A1	
	(ii) $R - mg\cos\theta = \frac{mv^2}{r}$	M1 A1	
	$R = \frac{m}{r}gr(1 + 2\cos\theta) + mg\cos\theta$ $\quad R = mg(1 + 3\cos\theta)$	M1 A1	
	(iii) (a) When $v = 0$, $\cos\theta = -\frac{1}{2}$ $\quad$ (b) When $R = 0$, $\cos\theta = -\frac{1}{3}$	B1 B1	11
7.	(i) $mv\frac{dv}{dx} = -\frac{mv^2}{k+x}$ $\quad\int\frac{dv}{v} = -\int\frac{dx}{k+x}$ $\quad\ln v = c - \ln(k + x)$	M1 A1 A1	
	$v = u, x = 0$: $c = \ln u + \ln k = \ln ku$ $\quad\ln v = \ln ku - \ln(k + x)$	M1 A1	
	$v = \frac{ku}{k+x}$	A1	
	(ii) $v = \frac{dx}{dt} = \frac{ku}{k+x}$ $\quad\int(k + x)\,dx = \int ku\,dt$ $\quad kx + \frac{x^2}{2} = kut + c$	M1 A1	
	$x = 0, t = 0$: $c = 0$ $\quad x = 2, t = 2$: $2k + 2 = 2ku$	A1 M1	
	$k = \frac{1}{u-1}$ $\quad k > 0$, so $u > 1$	A1 A1	12

