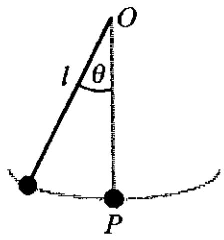
MECHANICS (C) UNIT 3 **TEST PAPER 3**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. The diagram shows a simple pendulum of length l m at the instant when its angular displacement from the equilibrium position OP is θ radians.



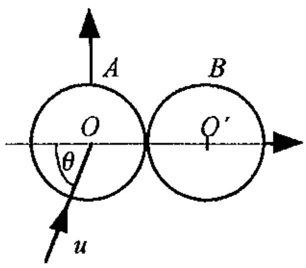
The equation of motion of the pendulum is given by $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$.

Given that θ is small,

(i) show that the motion is simple harmonic. [2]

(ii) Write down the period of the motion. [2]

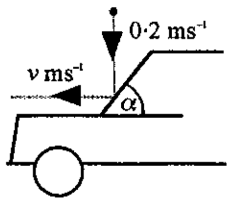
2. The diagram shows two smooth, perfectly elastic spheres A and B, of masses m and M respectively. Initially B is at rest and A is moving with speed u in a direction making an angle θ with the line of centres OO'. The spheres collide, and after the impact, A moves perpendicular to OO' and B moves parallel to OO'.



(i) Find, in terms of u and θ, the speeds of A and B after the impact. [3]

(ii) Show that $M = m$. [4]

3. A hailstone falling vertically with speed $0{\cdot}2$ ms^{-1} strikes the windscreen of a car and rebounds horizontally with speed v ms^{-1} as shown. Modelling the hailstone as a particle and the windscreen as a smooth plane inclined at an angle α to the horizontal,



(i) show that $v = 0{\cdot}2 \tan \alpha$. [3]

Given also that $\tan \alpha = \frac{3}{4}$,

(ii) find the coefficient of restitution between the hailstone and the windscreen. [5]

4. A light elastic string, of natural length $0{\cdot}8$ m, has one end fastened to a fixed point O. The other end of the string is attached to a particle P of mass $0{\cdot}5$ kg. When P hangs in equilibrium, the length of the string is $1{\cdot}5$ m.

(i) Calculate the modulus of elasticity of the string. [3]

P is displaced to a point $0{\cdot}5$ m vertically below its equilibrium position and released from rest.

(ii) Show that the subsequent motion of P is simple harmonic, with period $1{\cdot}68$ s. [4]

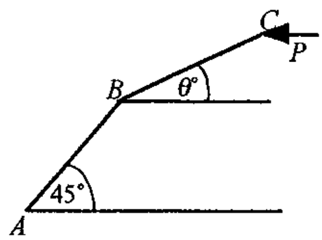
(iii) Calculate the maximum speed of P during its motion. [2]

5. A particle P of mass 0·4 kg hangs by a light, inextensible string of length 20 cm whose other end is attached to a fixed point O. It is given a horizontal velocity of 1·4 ms^{-1} so that it begins to move in a vertical circle. If, in the ensuing motion, the string makes an angle of θ with the downward vertical through O, show that
   (i) θ cannot exceed 60°, [6]
   (ii) the tension, T N, in the string is given by $T = 3{\cdot}92(3\cos\theta - 1)$. [4]

6. A particle P of mass m kg moves vertically upwards under gravity, starting from ground level. It is acted on by a resistive force of magnitude $m\,\text{f}(x)$ N, where f(x) is a function of the height x m of P above the ground. When P is at this height, its upward speed v ms^{-1} is given by $v^2 = 2e^{-2gx} - 1$.
   (i) Write down a differential equation for the motion of P and hence determine f(x) in terms of g and x. [5]
   (ii) Show that the greatest height reached by P above the ground is $\frac{1}{2g}\ln 2$ m. [2]

   Given that the work, in J, done by P against the resisting force as it moves from ground level to a point H m above the ground is equal to $\int_0^H m\,\text{f}(x)\,\text{d}x$,
   (iii) show that the total work done by P against the resistance during its upward motion is $\frac{1}{2}m(1 - \ln 2)$ J. [3]

7. Two identical uniform rods AB and BC, each of weight mg, are freely jointed at B. The end A is smoothly hinged to a fixed point. The system is kept in equilibrium in a vertical plane by a horizontal force of magnitude P applied at C, and the rods then make angles 45° and θ° with the horizontal as shown.



   (i) Write down the magnitude of the vertical component of the force acting on AB at A, and show that the horizontal component of this force has magnitude $\frac{3mg}{2}$. [5]
   (ii) Hence state, with reasons, the magnitudes of the horizontal and vertical components of the force acting on BC at B. [3]
   (iii) Explain why $P = \frac{3mg}{2}$. [1]
   (iv) Show that $\tan\theta = \frac{1}{3}$. [3]

MECHANICS 3 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1.	(i) θ small, so $\sin\theta \approx \theta$ Hence $\frac{d^2\theta}{dt^2} \approx -\frac{g}{l}\theta$; acc. proportional to	M1 A1	
	(angular) displacement, so SHM. (ii) Period $= 2\pi/\sqrt{(g/l)} = 2\pi\sqrt{\frac{l}{g}}$	M1 A1	4
2.	(i) Cons. of mom. $\perp OO'$: $v_A = u\sin\theta$ Restitution : $v_B = u\cos\theta$	B1 M1 A1	
	(ii) Cons. of momentum // OO': $Mv_B = mu\cos\theta$ $v_B = \frac{mu}{M}\cos\theta$	M1 A1	
	Now $e = 1$, so $\frac{mu}{M} = m$ $M = m$	M1 A1	7
3.	(i) Mom. // to plane : $m(0{\cdot}2\sin\alpha) = m(v\cos\alpha)$ $v = 0{\cdot}2\tan\alpha$	M1 A1 A1	
	(ii) Restitution $\perp$ to plane : $e(0{\cdot}2\cos\alpha) = v\sin\alpha$	M1 A1	
	$e = \frac{0{\cdot}2\tan\alpha\sin\alpha}{0{\cdot}2\cos\alpha} = \tan^2\alpha = \frac{9}{16}$	M1 A1 A1	8
4.	(i) $mg = \frac{\lambda}{0{\cdot}8} \times 0{\cdot}7 = 0{\cdot}5 \times 9{\cdot}8$ $\lambda = 4{\cdot}9 \times \frac{0{\cdot}8}{0{\cdot}7} = 5{\cdot}6$ N	M1 A1 A1	
	(ii) $(0{\cdot}5 \times 9{\cdot}8) - \frac{5{\cdot}6}{0{\cdot}8}(0{\cdot}7 + x) = 0{\cdot}5\ddot{x}$ $4{\cdot}9 - 4{\cdot}9 - 7x = 0{\cdot}5\ddot{x}$	M1 A1	
	$\ddot{x} = -14x$, of form $\ddot{x} = n^2x$ with $n^2 = 14$, so simple harmonic	A1	
	Period $= 2\pi/\sqrt{14} = 1{\cdot}68$ s	A1	
	(iii) Maximum speed $= an = 0{\cdot}5\sqrt{14} = 1{\cdot}87$ ms^{-1}	M1 A1	9
5.	(i) Energy : $\frac{1}{2}(0{\cdot}4)(1{\cdot}4)^2 = 0{\cdot}4 \times 9{\cdot}8 \times 0{\cdot}2(1 - \cos\theta) + \frac{1}{2} \times 0{\cdot}4v^2$	M1 A1 A1	
	$v^2 = 1{\cdot}96 - 3{\cdot}92(1 - \cos\theta) = 3{\cdot}92\cos\theta - 1{\cdot}96$	A1	
	$v^2 \geq 0$, so $\cos\theta \geq \frac{1}{2}$ $\theta \leq 60°$	M1 A1	
	(ii) $T - mg\cos\theta = \frac{mv^2}{r}$ $T = 0{\cdot}4 \times 9{\cdot}8 \times \cos\theta + 2(3{\cdot}92\cos\theta - 1{\cdot}96)$	B1 M1 A1	
	$T = 3{\cdot}92(3\cos\theta - 1)$	A1	10
6.	(i) $mv\frac{dv}{dx} = -(mg + mf(x))$ $v\frac{dv}{dx} = -g - f(x)$	M1 A1	
	$v^2 = 2e^{-2gx} - 1$, so $2v\frac{dv}{dx} = -4ge^{-2gx}$ $-2ge^{-2gx} = -g - f(x)$	M1 A1	
	$f(x) = g(2e^{-2gx} - 1)$	A1	
	(ii) $v = 0$ when $2e^{-2gx} = 1$ $x = \frac{1}{2g}\ln 2$	M1 A1	
	(iii) W.D. $= m[-e^{-2gx} - gx]_0^{(\ln 2)/2g} = m(-e^{-\ln 2} - \frac{1}{2}\ln 2 + 1) = \frac{1}{2}m(1 - \ln 2)$	M1 A1 A1	10
7.	(i) Vertical comp. $Y = 2mg$ Let $AB = 2l$	B1	
	M(B) for AB: $X\frac{2l}{\sqrt{2}} + mg\frac{l}{\sqrt{2}} = 2mg\frac{2l}{\sqrt{2}}$ $X = \frac{3}{2}mg$	M1 A1 A1 A1	
	(ii) Separating rods at B, let horizontal and vertical components of	M1	
	contact force be X_1 and Y_1 Then AB (hor.) gives $X_1 = X = \frac{3}{2}mg$	A1	
	and BC (vert.) gives $Y_1 = mg$	A1	
	(iii) Horizontal forces on BC give $P = X_1 = \frac{3}{2}mg$	A1	
	(iv) M(B) for BC : $\frac{3}{2}mg(2l\sin\theta) = mg(l\cos\theta)$	M1 A1	
	$3\sin\theta = \cos\theta$ $\tan\theta = \frac{1}{3}$	A1	12

