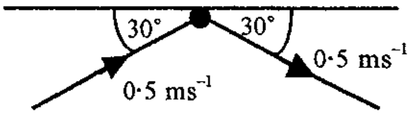
MECHANICS (C) UNIT 3 TEST PAPER 2

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. The diagram shows a smooth billiard ball, of mass 0·6 kg, approaching and rebounding from the side of a horizontal table. Both before and after the impact, it has speed 0·5 ms^{-1} and its direction of motion makes an angle of 30° with the side of the table.



Find the magnitude and direction of the impulse on the ball due to the impact with the table. [4]

2. A particle P of mass m kg is attached to one end of a light elastic string of natural length l m and modulus of elasticity λ N. The other end of the string is attached to a fixed point O. P hangs in equilibrium at the point E where $OE = \frac{5l}{4}$ m.

   (i) Show that $\lambda = 4mg$. [2]

   P is pulled down vertically to A, where $OA = 2l$ m, and released.

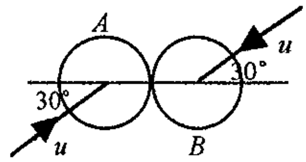
   (ii) Find, in terms of l and g, the speed with which P passes through E. [4]

3. A particle moves along a straight line in such a way that its displacement x m from a fixed point O on the line, at time t seconds after it leaves O, is given by $x = p \sin \omega t + q \cos \omega t$ where p, q and ω are constants.

   (i) Show that the motion of the particle is simple harmonic. [5]

   (ii) If the particle leaves O with speed 15 ms^{-1}, and $\omega = 3$, find the amplitude of the motion. [2]

4. Two identical smooth spheres A and B, each of mass m kg, are moving in opposite directions along parallel lines with equal speeds u ms^{-1}. They collide obliquely, and at the moment of impact their directions of motion make an angle of 30° with the line joining their centres, as shown.



   (i) State, in terms of m and u, the total kinetic energy of the two spheres before the impact. [1]

   Given that the coefficient of restitution between A and B is e,

   (ii) show that, after impact, the components of B's speed along and perpendicular to the line of centres are $\frac{eu\sqrt{3}}{2}$ ms^{-1} and $\frac{u}{2}$ ms^{-1} respectively. [6]

   (iii) Hence calculate, in terms of e, u and m, the loss in kinetic energy due to the impact. [3]

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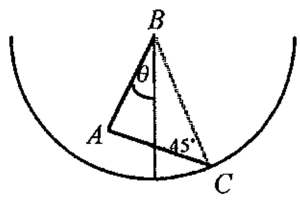
5. A particle P is attached to one end of a light inextensible string of length l m. The other end of the string is attached to a fixed point O. When P is hanging at rest vertically below O, it is given a horizontal speed u ms^{-1} and starts to move in a vertical circle.
Given that the string becomes slack when it makes an angle of 120° with the downward vertical through O, show that $u^2 = \frac{7gl}{2}$. [10]

6. Two uniform rods AB and AC, each of length a and weight mg, are smoothly jointed at A. B is freely hinged to the centre of a hollow sphere and C rests against the rough interior surface of the sphere with angle $BAC = 90°$, angle $ACB = 45°$ and AB inclined at an angle θ to the vertical. A, B and C are in a vertical plane.



In this position, C is on the point of slipping down the surface.

(i) By taking moments about B for the system consisting of the two rods, or otherwise, show that the magnitude of the frictional force at C is $\frac{mg}{2\sqrt{2}}(\cos\theta - 3\sin\theta)$. [5]

(ii) Find, in a similar form, the magnitude of the normal reaction at C. [4]

Given that the coefficient of friction between C and the surface of the sphere is $\frac{1}{3}$,

(iii) show that $\theta = 9{\cdot}5°$. [2]

7. A particle of mass m kg is projected vertically upwards with speed u ms^{-1} In addition to its weight, the only other force acting on it is a resistance of magnitude $mgkv^2$ N opposing its motion, where k is a positive constant and v ms^{-1} is the speed of the particle. Show that

(i) the greatest height reached by the particle above the point of projection is
$\frac{1}{2kg}\ln(1 + ku^2)$ m, [6]

(ii) the particle returns to its point of projection with speed $\frac{u}{\sqrt{(1+ku^2)}}$ ms^{-1}. [6]

MECHANICS 3 (C) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1.	$\mathbf{J} = m\mathbf{v} - m\mathbf{u}$ Recognising equilateral Δ, or by calculation	B1 M1	
	$\lvert\mathbf{J}\rvert = 0{\cdot}6 \times 0{\cdot}5 = 0{\cdot}3$ Ns, perpendicular to side of table	A1 A1	4
2.	(i) At E, in equilibrium, $mg = \frac{\lambda}{l}\cdot\frac{l}{4}$ $\lambda = 4mg$	M1 A1	
	(ii) Elastic P.E. gained = gravitational P.E. lost + K.E. gained		
	$\frac{4mg}{2l}\left(l^2 - \frac{l^2}{16}\right) = mg\frac{3l}{4} + \frac{1}{2}mv^2$ $\frac{15gl}{8} - \frac{3gl}{4} = \frac{1}{2}v^2$ $v = \frac{3}{2}\sqrt{gl}$	M1 A1 A1 A1	6
3.	(i) $x = p\sin\omega t + q\cos\omega t$ $\dot{x} = p\omega\cos\omega t - q\omega\sin\omega t$	M1 A1	
	$\ddot{x} = -p\omega^2\sin\omega t - q\omega^2\cos\omega t = -\omega^2(p\sin\omega t + q\cos\omega t) = -\omega^2 x$	M1 A1	
	Acc. prop. to displacement and directed towards O, so SHM	B1	
	(ii) Maximum speed at O, so $15 = a\omega = 3a$ $a = 5$ m	M1 A1	7
4.	(i) Total K.E. $= \frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2$ J	B1	
	(ii) // line of centres : $mv_A + mv_B = 0$, $(v_A - v_B)/(2u\cos 30°) = -e$	M1 A1	
	Hence $v_B = ue\cos 30° = \frac{eu\sqrt{3}}{2}$	M1 A1	
	⊥ line of centres, speed unchanged; component $= u\sin 30° = \frac{u}{2}$	M1 A1	
	(iii) New K.E. of either sphere $= \frac{1}{2}m\left(\frac{3e^2u^2}{4} + \frac{u^2}{4}\right)$	B1	
	K.E. lost $= mu^2 - m\left(\frac{3e^2u^2}{4} + \frac{u^2}{4}\right) = \frac{3mu^2}{4}\left(1 - e^2\right)$ J	M1 A1	10
5.	K.E. lost = P.E. gained : $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl - mgl\cos\theta$	M1 A1	
	$v^2 = u^2 - 2gl + 2gl\cos\theta$	A1	
	$T - mg\cos\theta = \frac{mv^2}{l}$ $T = \frac{m}{l}(u^2 - 2gl + 2gl\cos\theta) + mg\cos\theta$	M1 A1 M1 A1	
	$T = 0$ when $\theta = 120°$: $\frac{u^2}{l} - 2g - g - \frac{g}{2} = 0$ $u^2 = \frac{7gl}{2}$	M1 A1 A1	10
6.	(i) M(B) : $mg\frac{a}{2}\sin\theta + Fa\sqrt{2} = mg\left(\frac{a}{2}\cos\theta - a\sin\theta\right)$	M1 A1	
	$mg\sin\theta + 2F\sqrt{2} = mg\cos\theta - 2mg\sin\theta$ $F = \frac{mg}{2\sqrt{2}}(\cos\theta - 3\sin\theta)$	M1 A1 A1	
	(ii) M(A) for AC : $R\frac{a}{\sqrt{2}} = mg\frac{a}{2}\cos\theta - F\frac{a}{\sqrt{2}}$	M1 A1	
	$R = \frac{2}{2\sqrt{2}}\cos\theta - F = \frac{mg}{2\sqrt{2}}(\cos\theta + 3\sin\theta)$	M1 A1	
	(iii) $F = \mu R$: $\cos\theta + 3\sin\theta = 3(\cos\theta - 3\sin\theta)$ $6\tan\theta = 1$ $\theta = 9{\cdot}5°$	M1 A1	11
7.	(i) Rising, $mv\frac{dv}{dx} = -mg - mgkv^2$ $\int \frac{v}{1+kv^2}\,dv = -\int g\,dx$	M1 A1	
	$\frac{1}{2k}\ln(1 + kv^2) = -gx + c$ $x = 0, v = u$, so $c = \frac{1}{2k}\ln(1 + ku^2)$	M1 A1	
	When $v = 0$, $gx = c$ $x = \frac{1}{2kg}\ln(1 + ku^2)$	M1 A1	
	(ii) Falling, $mv\frac{dv}{dx} = -mg + mgkv^2$ $-\frac{1}{2k}\ln(1 - kv^2) = -gx + c'$	M1 A1	
	$c' = c$; when $x = 0$, $\ln(1 - kv^2) = -\ln(1 + ku^2)$	M1 A1	
	$1 - kv^2 = \frac{1}{1+ku^2}$ $kv^2 = \frac{ku^2}{1+ku^2}$ $v = \frac{u}{\sqrt{1+ku^2}}$	M1 A1	12