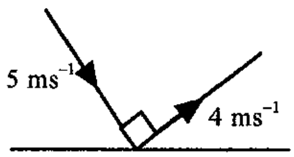
MECHANICS (C) UNIT 3 **TEST PAPER 1**

Take $g = 9{\cdot}8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

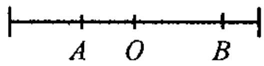
1. A particle P of mass m kg is attached to the mid-point of a light elastic string of natural length $8l$ m and modulus of elasticity λ N. The two ends of the string are attached to fixed points A and B on the same horizontal level, where $AB = 8l$ m. P is released from rest at the mid-point of AB.

   If P comes to instantaneous rest at a depth $3l$ m below AB, find an expression for λ in terms of m and g. [4]

2. A particle of mass $0{\cdot}2$ kg is travelling at a speed of 5 ms^{-1} when it hits the ground and rebounds with speed 4 ms^{-1} at right angles to its original direction of motion, as shown.
   Assuming that there is no friction between the particle and the ground, calculate the magnitude and direction of the impulse on the particle due to the impact. [5]

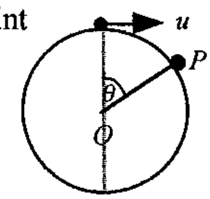


3. A particle P moves with simple harmonic motion in a straight line, with the centre of motion at the point O on the line. A and B are on opposite sides of O, with $OA = 4$ m, $OB = 6$ m.
   When passing through A and B, P has speed 6 ms^{-1} and 4 ms^{-1} respectively.



   (i) Find the amplitude of the motion. [5]
   (ii) Show that the period of motion is 2π seconds. [2]

4. A particle P is projected horizontally with speed $u \text{ ms}^{-1}$ from the highest point of a smooth sphere of radius r m and centre O. It moves on the surface in a vertical plane, and at a particular instant the radius OP makes an angle θ with the upward vertical, as shown. At this instant P has speed $v \text{ ms}^{-1}$ and the magnitude of the reaction between P and the sphere is X N.

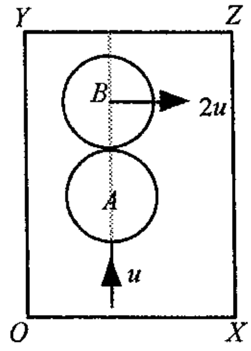


   (i) Assuming that $u^2 < gr$, show that (a) $v^2 = u^2 + 2gr(1 - \cos\theta)$, [3]

   (b) $X = mg\left(3\cos\theta - 2 - \frac{u^2}{gr}\right)$. [4]

   (ii) Show that P leaves the surface of the sphere when $\cos\theta = \dfrac{u^2 + 2gr}{3gr}$. [2]

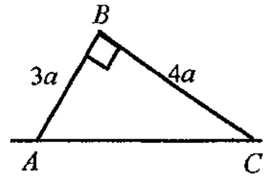
5. Two identical smooth spheres A and B are free to move on a smooth horizontal table $OXZY$. Initially A and B are moving with speeds u and $2u$ parallel to OY and OX respectively, as shown. At the moment of impact, the line of centres of the spheres is parallel to OY. The coefficient of restitution between the spheres is e.



   (i) Find, in terms of u, the components parallel to OX and OY of the velocity of A immediately after the impact. [9]

   (ii) Show that the direction of motion of B immediately after the impact makes an angle θ with OX, where $\tan\theta = \dfrac{1+e}{4}$. [2]

6. Two uniform rods, AB of length $3a$ and weight $2mg$ and BC of length $4a$ and weight mg, are smoothly jointed at B. They rest in equilibrium as shown, with the plane ABC vertical. A and C are in contact with a rough horizontal plane and angle $ABC = 90°$.



   (i) By taking moments about A, or otherwise, show that the vertical reaction at C has magnitude $\dfrac{26mg}{25}$. [4]

   (ii) Find the magnitude of the horizontal frictional force at C. [4]

   The force acting on AB at B has horizontal and vertical components of magnitudes X and Y respectively.

   (iii) Show that $X = 18Y$. [3]

7. A skier of mass m kg starts from rest and races down a smooth ramp inclined at 30° to the horizontal. He encounters air resistance, which at time t s after he starts is proportional to his speed v ms^{-1}.

   Given that the maximum speed he reaches on the ramp is u ms^{-1},

   (i) show that $\dfrac{dv}{dt} = \dfrac{g}{2u}(u - v)$. [3]

   (ii) Hence prove that $v = u(1 - e^{-gt/2u})$. [7]

   The skier reaches half his maximum speed at time T s after he starts.

   (iii) Find T in the form $\dfrac{ku}{g}$, stating the value of the constant k. [3]

MECHANICS 3 (C) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1.	Grav. P.E. loss = E.P.E. gain : $3mgl = \frac{\lambda}{16l}(10l - 8l)^2$ $\lambda = 12\,mg$	M1 A1 M1 A1	4
2.	Impulse vector $\mathbf{J} = m\mathbf{v} - m\mathbf{u}$ $\lvert \mathbf{J} \rvert = \sqrt{(1^2 + 0{\cdot}8^2)} = 1{\cdot}28$ Ns	B1 M1 A1	
	Momentum along ground is conserved, so direction of **J** is vertical	M1 A1	5
3.	(i) $v^2 = n^2(a^2 - x^2)$ $36 = n^2(a^2 - 16),\ 16 = n^2(a^2 - 36)$	M1 A1 A1	
	$36(a^2 - 36) = 16(a^2 - 16)$ $20a^2 = 1040$ $a = 7{\cdot}21$ m	M1 A1	
	(ii) $n^2 = 1$ $n = 1$ Period $= \frac{2\pi}{n} = 2\pi$ s	M1 A1	7
4.	(i) (a) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgr(1 - \cos\theta)$ $v^2 = u^2 + 2gr(1 - \cos\theta)$	M1 A1 A1	
	(b) $mg\cos\theta - X = \frac{mv^2}{r}$ $X = mg\cos\theta - \frac{mu^2}{r} - 2mg(1 - \cos\theta)$	M1 A1 A1	
	$X = mg[3\cos\theta - 2 - \frac{u^2}{gr}]$	A1	
	(ii) Leaves sphere when $X = 0$, i.e. when $3\cos\theta = 2 + \frac{u^2}{gr}$, etc.	M1 A1	9
5.	(i) Cons. of mom. // OX : components are 0 for A, $v = 2u$ for B	B1 B1	
	Let final components of velocity // OY be v for A, w for B		
	Cons. of momentum // OY : $mu = mv + mw$ $u = v + w$	M1 A1	
	Restitution : $v - w = -eu$ Solve : $v = \frac{1-e}{2}u,\ w = \frac{1+e}{2}u$	M1 A1 M1 A1 A1	
	(ii) For B, $\tan\theta = \frac{w}{2u} = \left(\frac{1+e}{2}u\right)\frac{1}{2u} = \frac{1+e}{4}$	M1 A1	11
6.	(i) M(A) : $R(5a) = mg\left(5a - \frac{8a}{5}\right) + 2mg\left(\frac{9a}{10}\right)$ $R = \frac{26mg}{25}$	M1 A1 A1 A1	
	(ii) M(B) for BC : $F.4a.\frac{3}{5} + mg.2a.\frac{4}{5} = \frac{26mg}{25}.4a.\frac{4}{5}$ $F = \frac{18mg}{25}$	M1 A1 M1 A1	
	(iii) $X = F = \frac{18mg}{25}$ $Y = \frac{26mg}{25} - mg = \frac{mg}{25}$ $X = 18Y$	B1 M1 A1	11
7.	(i) $m\frac{dv}{dt} = \frac{mg}{2} - kv$ At max. speed, $0 = \frac{mg}{2} - ku$ $k = \frac{mg}{2u}$	M1 A1	
	Hence $\frac{dv}{dt} = \frac{g}{2u}(u - v)$	A1	
	(ii) $\int \frac{1}{u-v}\,dv = \int \frac{g}{2u}\,dt$ $-\ln(u - v) = \frac{g}{2u}t + c$	M1 A1	
	$v = 0, t = 0 : c = -\ln u$ $-\ln(u - v) = \frac{g}{2u}t - \ln u$	M1 A1	
	$\ln\left(\frac{u-v}{u}\right) = -\frac{gt}{2u}$ $\frac{u-v}{u} = e^{-\frac{gt}{2u}}$ $v = u\left(1 - e^{-\frac{gt}{2u}}\right)$	M1 A1 A1	
	(iii) $v = \frac{u}{2}$: $\frac{1}{2} = 1 - e^{-\frac{gT}{2u}}$ $\frac{gT}{2u} = \ln 2$ $T = \frac{2u}{g}\ln 2$ $k = 2\ln 2$	M1 A1 A1	13