MECHANICS 3 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $R\cos 8° = mg$, $R\sin 8° = \frac{mv^2}{10}$ $\tan 8° = \frac{v^2}{9{\cdot}8\times 10}$ M1 A1 M1 A1
   $\frac{v^2}{98} = \tan 8°$ $v = 3{\cdot}71$ ms^{-1} M1 A1 6

2. $APB = 90°$ $\sin\theta = 0{\cdot}6, \cos\theta = 0{\cdot}8$ $T\cos\theta + S\sin\theta = 0{\cdot}8g$ B1 M1
   $4T + 3S = 39{\cdot}2$ Horiz : $S\cos\theta = T\sin\theta$, so $4S = 3T$ A1 B1
   Solve : $S = 4{\cdot}704$ N, $T = 6{\cdot}272$ N Now $S = \frac{\mu}{0{\cdot}2}\times 0{\cdot}2 = \mu$, M1 A1 (both)
   $T = \frac{\lambda}{0{\cdot}2}\times 0{\cdot}1 = \frac{\lambda}{2}$ So $\lambda = 12{\cdot}5$, $\mu = 4{\cdot}70$ M1 A1 A1 9

3. (a) $\frac{dv}{dt} = -v^2\sin\left(\frac{t}{100}\right)$ $\int \frac{1}{v^2}\,dv = -\int \sin\left(\frac{t}{100}\right)dt$ M1 A1
   $-\frac{1}{v} = 100\cos(t/100) + c$ $t = 0, v = 0{\cdot}2 : c = -105$ A1 M1 A1
   $\frac{1}{v} = 105 - 100\cos\left(\frac{t}{100}\right)$ $v = \frac{1}{105-100\cos\left(\frac{t}{100}\right)}$ M1 A1
   (b) $v_{max} = 0{\cdot}2$ ms^{-1} (initial speed) $v_{min} = 0{\cdot}00952$ ms^{-1} $(t = 50\pi)$ M1 A1 A1 10

4. (a) $\bar{x}\int_0^\pi y\,dx = \int_0^\pi xy\,dx$ $\bar{x}\int_0^\pi 1+\cos x\,dx = \int_0^\pi x + x\cos x\,dx$ M1 A1 A1
   $\bar{x}[x+\sin x]_0^\pi = [\frac{1}{2}x^2 + x\sin x + \cos x]_0^\pi$ (R.H.S. by parts) M1 A1 A1 A1
   $\pi\bar{x} = \frac{\pi^2}{2} - 2$ $\bar{x} = \frac{\pi^2-4}{2\pi}$ M1 A1
   (b) $\tan\theta = \frac{\pi^2-4}{2\pi}\cdot\frac{4}{5} = 0{\cdot}7473$ $\theta = 36{\cdot}8°$ M1 A1 A1 12

5. (a) $T = F = \mu R$, so $T = \frac{1}{4}g$ $T\cos\theta = 0{\cdot}2g$ B1 M1 A1
   $\cos\theta = 0{\cdot}8$ $\theta = 36{\cdot}9°$ A1
   (b) $T\sin\theta = 0{\cdot}2v^2/(0{\cdot}4\sin\theta)$ $v^2 = 0{\cdot}5g\sin^2\theta = 1{\cdot}764$ M1 A1
   $v = \sqrt{1{\cdot}764} = 1{\cdot}33$ ms^{-1} A1
   (c) Now $T = 0{\cdot}2g$ $0{\cdot}2g + 0{\cdot}25g = 0{\cdot}5\frac{v^2}{r}$ B1 M1 A1
   $0{\cdot}45g = 0{\cdot}5(0{\cdot}84^2)/r$ $r = 0{\cdot}08$ M1 A1 12

6. (a) $24g = 2T = 2\frac{\lambda}{l}(0{\cdot}3)$ $\frac{\lambda}{l} = \frac{24\times 9{\cdot}8}{2\times 0{\cdot}3} = 392$ $\lambda = 392l$ M1 A1
   (b) At dist. x from A, $mg - 2\frac{\lambda}{l}(0{\cdot}3 + x) = m\ddot{x}$ M1 A1 A1
   $\ddot{x} = -\frac{2l}{ml}x = -\frac{98}{3}x$ Hence S.H.M. with centre A A1 A1
   (c) $\omega^2 = \frac{98}{3} = 32{\cdot}7$ Freq. $= \frac{\omega}{2\pi} = \frac{\sqrt{32{\cdot}7}}{2\pi} = 0{\cdot}91$ osc. s^{-1} M1 A1 A1
   (d) Max. acc. $= \omega^2(0{\cdot}2) = 6{\cdot}54$ ms^{-2} M1 A1 12

7. (a) Radius of vert. circle $= 0{\cdot}4\sin\theta$ At top, $T = 0$ (just) B1 B1
   $2mg(0{\cdot}4\sin\theta) + \frac{1}{2}mv^2 = \frac{1}{2}mu^2$ $T + mg = \frac{mv^2}{0{\cdot}4\sin\theta}$ M1 A1 M1 A1
   $v^2 = 0{\cdot}4g\sin\theta$ $u^2 = 2g\sin\theta$ A1 A1
   (b) In subsequent horizontal motion, with tension S in string, B1
   $S\sin\theta = mg$, $S\cos\theta = \frac{mu^2}{0{\cdot}4\cos\theta} = \frac{2mg\sin\theta}{0{\cdot}4\cos\theta}$ M1 A1 A1
   Hence $S\cos^2\theta = 5S\sin^2\theta$ $\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{5}$ $\tan\theta = \frac{1}{5}\sqrt{5}$ M1 A1 14