MECHANICS 3 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. (a) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$ $v = 0, h = 2r : \frac{1}{2}u^2 = 2gr$ $u = 2\sqrt{(gr)}$ M1 A1 A1

   (b) At top, force towards centre $= \frac{mv^2}{r}$, $= mg$ as $R = 0$ M1 A1

   Thus $v^2 = gr$, so $mu^2 = mv^2 + 4mgr = 5mgr$ $u = \sqrt{(5gr)}$ M1 A1 7

2. Height of each section $= \frac{3h}{24} = \frac{h}{8}$, mass of each section $= \frac{3M}{24} = \frac{M}{8}$ B1 B1

   M(BD): $\frac{M}{4}\left(\frac{h}{8}\right) + \frac{M}{8}\left(\frac{7h}{16}\right) + \frac{M}{8}\left(\frac{11h}{16}\right) + \frac{M}{8}\left(\frac{15h}{16}\right) = \frac{5M}{8}\bar{y}$ M1 A1 A1

   $\frac{5}{8}\bar{y} = \frac{37}{128}h$ $\bar{y} = \frac{37}{80}h\ (\approx 0{\cdot}46h)$ M1 A1 7

3. (a) Loss in P.E. = gain in E.P.E. : $Mge = \frac{\lambda}{2l}e$ $e = \frac{2Mgl}{\lambda}$ M1 A1 A1

   (b) Loss in P.E. of A = gain in K.E. of (A & B) + gain in E.P.E. M1 M1

   $3 \times 9{\cdot}8 \times 0{\cdot}5 = \frac{1}{2}(4{\cdot}5)v^2 + 17{\cdot}5(0{\cdot}25)$ $v^2 = 4{\cdot}589$ $v = 2{\cdot}14\ \text{ms}^{-1}$ A1 A1 M1 A1 9

4. (a) $mv\frac{dv}{dx} = -\frac{km}{x^2}$ $\frac{v^2}{2} = \frac{k}{x} + c$ $x = a, v = 0$; $c = -\frac{k}{a}$ M1 A1 M1 A1

   $\frac{v^2}{2} = k\left(\frac{1}{x} - \frac{1}{a}\right)$ $v = \sqrt{\frac{2k}{a}\left(\frac{a-x}{x}\right)}$ M1 A1

   (b) $v = \frac{dx}{dt}$, so $\frac{dx}{dt} = \sqrt{\frac{2k(a-x)}{ax}} = \sqrt{\frac{1-x}{x}}$ $\int dt = \int\sqrt{\frac{x}{1-x}}\,dx$ M1 A1 A1

   $T = [\arcsin(\sqrt{x}) - \sqrt{(x - x^2)}]_{1/2}^{1} = \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi+2}{4}$ M1 A1 11

5. (a) With no slip, $R\sin 4° = \frac{mu^2}{100}$, $R\cos 4° = mg$ B1 B1

   $u^2 = 100g\tan 4° = 68{\cdot}53$ $u = 8{\cdot}28$ M1 A1

   (b) $R\cos 4° - F\sin 4° = mg$ $R\sin 4° + F\cos 4° = \frac{m(12{\cdot}5)^2}{100}$ M1 A1 M1 A1

   Solve : $R = 9{\cdot}885m$, $F = 0{\cdot}875m$ $F \le \mu R$, so $\mu \ge F/R = 0{\cdot}089$ M1 A1 A1 M1 A1 13

6. (a) Reaction R acts on P towards centre of sphere, at θ to vertical M1

   where $\cos\theta = \frac{h}{r}$ $R\cos\theta = mg$, so $R = \frac{mgr}{h}$ B1 M1 A1

   (b) Resolve towards centre : $R\sin\theta = \frac{mv^2}{r\sin\theta}$ B1

   $v^2 = \frac{mgr^2}{h}\frac{\sin^2\theta}{m} = \frac{gr^2}{h}\frac{(r^2-h^2)}{r^2}$ $v = \sqrt{\frac{g(r^2-h^2)}{h}}$ M1 A1 A1

   (c) $mg\sin\theta + S\cos\theta = mg$ $S = mg\frac{(1-\sin\theta)}{\cos\theta} = mg\left(1 - \frac{0{\cdot}866}{0{\cdot}5}\right)$ M1 A1 M1

   $= 0{\cdot}268mg$ $R = mg\frac{2h}{h} = 2mg$ Hence $S < \frac{R}{6}$ A1 A1 13

7. (a) Eqm. : $T = F$ $\frac{\lambda}{l}\cdot\frac{l}{4} = \mu mg$ $\mu = \frac{1}{4}$ M1 A1

   (b) At dist. x, $T - \mu mg = -m\ddot{x}$ $\frac{mg}{l}\left(\frac{l}{4} + x\right) - \frac{1}{4}mg = -m\ddot{x}$ M1 A1 A1

   $\ddot{x} = -\frac{g}{l}x$ S.H.M. with $\omega^2 = \frac{g}{l}$ A1

   (c) Amplitude $= \frac{3l}{4}$ $x = \frac{3l}{4}\cos\omega t$ $x = -\frac{l}{4}$: $t = \frac{1}{\omega}\arccos\left(-\frac{1}{3}\right)$ M1 A1 M1

   $= \sqrt{\frac{l}{g}}\left(\frac{\pi}{2} + \arcsin\left(\frac{1}{3}\right)\right)$ s M1 A1

   (d) At nat. length, $v^2 = \frac{g}{l}\left(\frac{9l^2}{16} - \frac{l^2}{16}\right) = \frac{gl}{2}$ At O, $v^2 = \frac{gl}{2} - 2\frac{g}{4}$ $v = 0$ M1 A1 M1 A1 15

