

MECHANICS (A) UNIT 3**TEST PAPER 8**

Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A particle P of mass m kg moves in a horizontal circle at one end of a light elastic string of natural length l m and modulus of elasticity mg N. The other end of the string is attached to a fixed point O . Given that the string makes an angle of 60° with the vertical,
- (a) show that $OP = 3l$ m. (4 marks)
- (b) Find, in terms of l and g , the angular speed of P . (4 marks)

2. A particle P of mass m kg moves vertically upwards under gravity, starting from ground level. It is acted on by a resistive force of magnitude $m f(x)$ N, where $f(x)$ is a function of the height x m of P above the ground. When P is at this height, its upward speed $v \text{ ms}^{-1}$ is given by $v^2 = 2e^{-2gx} - 1$.

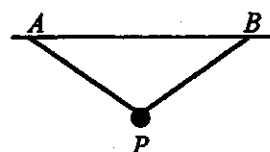
- (a) Write down a differential equation for the motion of P and hence determine $f(x)$ in terms of g and x . (5 marks)
- (b) Show that the greatest height reached by P above the ground is $\frac{1}{2g} \ln 2$ m. (2 marks)

Given that the work, in J, done by P against the resisting force as it moves from ground level to a point H m above the ground is equal to $\int_0^H m f(x) dx$,

- (c) show that the total work done by P against the resistance during its upward motion is $\frac{1}{2} m(1 - \ln 2)$ J. (3 marks)

3. A car of mass m kg moves round a curve of radius r m on a road which is banked at an angle θ to the horizontal. When the speed of the car is $u \text{ ms}^{-1}$, the car experiences no sideways frictional force. Given that $\tan \theta = \frac{u^2}{gr}$, show that the sideways frictional force on the car when its speed is $\frac{u}{2} \text{ ms}^{-1}$ has magnitude $\frac{3}{4} mg \sin \theta$ N. (10 marks)

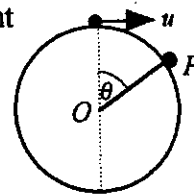
4. Two light elastic strings, each of length l m and modulus of elasticity λ N, are attached to a particle P of mass m kg. The other ends of the strings are attached to fixed points A and B on the same horizontal level, where $AB = 2l$ m. P is held vertically below the mid-point of AB , with each string taut and inclined at 30° to the horizontal, and released from rest.



- Given that P comes to instantaneous rest when each string makes an angle of 60° with the horizontal, show that $\lambda = \frac{3mg}{6 - 2\sqrt{3}}$. (10 marks)

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5. A particle P is projected horizontally with speed $u \text{ ms}^{-1}$ from the highest point of a smooth sphere of radius $r \text{ m}$ and centre O . It moves on the surface in a vertical plane, and at a particular instant the radius OP makes an angle θ with the upward vertical, as shown. At this instant P has speed $v \text{ ms}^{-1}$ and the magnitude of the reaction between P and the sphere is $X \text{ N}$.



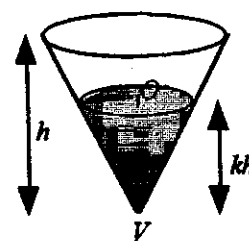
- (a) Assuming that $u^2 < gr$, show that
- (i) $v^2 = u^2 + 2gr(1 - \cos \theta)$, (2 marks)
- (ii) $X = mg\left(3\cos \theta - 2 - \frac{u^2}{gr}\right)$. (4 marks)
- (b) Show that P leaves the surface of the sphere when $\cos \theta = \frac{u^2 + 2gr}{3gr}$. (3 marks)
- (c) Discuss what happens if $u^2 \geq gr$. (2 marks)

6. A particle P of mass $m \text{ kg}$ hangs in equilibrium at one end of a light spring, of natural length $l \text{ m}$ and modulus of elasticity $\lambda \text{ N}$, whose other end is fixed at a point vertically above P . In this position the length of the spring is $(l + e) \text{ m}$. When P is displaced vertically through a small distance and released, it performs simple harmonic motion with 5 oscillations per second.

- (a) Show that $\frac{\lambda}{l} = 100\pi^2 m$. (8 marks)
- (b) Express e in terms of g . (2 marks)
- (c) Determine, in terms of m and l , the magnitude of the tension in the spring when it is stretched to twice its natural length. (2 marks)

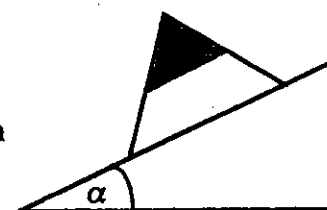
7. (a) Prove that the centre of mass of a uniform solid right circular cone of height h and base radius r is at a distance $\frac{3h}{4}$ from the vertex. (7 marks)

An item of confectionery consists of a thin wafer in the form of a hollow right circular cone of height h and mass m , filled with solid chocolate, also of mass m , to a depth of kh as shown. The centre of mass of the item is at O , the centre of the horizontal plane face of the chocolate.



- (b) Show that $k = \frac{8h}{15}$. (3 marks)

In the packaging process, the cone has to move on a conveyor belt inclined at an angle α to the horizontal as shown. If the belt is rough enough to prevent sliding, and the maximum value of α for which the cone does not topple is 45° ,



- (c) find the radius of the base of the cone in terms of h . (4 marks)