MECHANICS (A) UNIT 3 **TEST PAPER 5**

Take $g = 9{\cdot}8\ \text{ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A light spring, of natural length 30 cm, is fixed in a vertical position. When a small ball of mass 0·4 kg rests on top of it, the spring is compressed by 10 cm. The ball is then held at a height of 15 cm vertically above the top of the spring and released from rest.
Calculate the maximum compression of the string in the resulting motion. **(7 marks)**

2. Aliya, whose mass is m kg, is playing rounders. She rounds the first base at a speed of $v\ \text{ms}^{-1}$, making the turn on a horizontal circular path of radius r m.
(a) Write down, in terms of m, v and r, the magnitude of the horizontal force acting on her. **(1 mark)**
(b) Show that if she continues on the same circular path, the reaction force exerted on her by the ground must act at an angle θ to the vertical, where $\tan\theta = \frac{v^2}{gr}$. **(6 marks)**

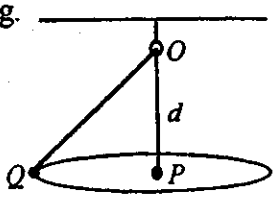
3. A particle P of mass 0·2 kg is suspended by two identical light inelastic strings, with one end of each string attached to P and the other ends fixed to points O and X on the same horizontal level. Both strings are inclined at 30° to the horizontal.
(a) Find the tension in the strings when P is at rest. **(2 marks)**
The string XP is suddenly cut, so that P begins to move in a vertical circle with centre O.
(b) Find the tension in the string OP when it makes an angle of 60° with the horizontal. **(6 marks)**

4. The radius of the Earth is R m. The force of attraction towards the centre of the Earth experienced by a body of mass m kg at a distance x m from the centre is of magnitude $\frac{km}{x^2}$ N, where k is a constant.
(a) Show that $k = gR^2$. **(1 mark)**

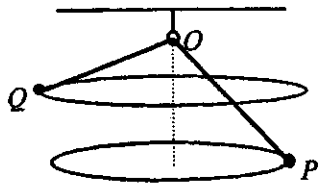
Two satellites A and B, each of mass m kg, are moving in circular orbits around the Earth at distances $3R$ m and $4R$ m respectively from the centre of the Earth. Given that the satellites move in the same plane and that they lie along the same radial line from the centre at any time,
(b) show that the ratio of the speed of B to that of A is 4 : 3. **(2 marks)**

If, in addition, the satellites are linked with a taut, straight wire in the same plane and along the same radial line,
(c) find, in terms of m and g, the magnitude of the force in the wire. **(8 marks)**

5. A light inelastic string of length l m passes through a small smooth ring which is fixed at a point O and is free to rotate about a vertical axis through O. Particles P and Q, of masses 0·06 kg and 0·04 kg respectively, are attached to the ends of the string.

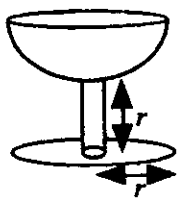


(a) Q describes a horizontal circle with centre P, while P hangs at rest at a depth d m below O. Show that $d = \frac{2l}{5}$. **(6 marks)**



(b) P and Q now both move in horizontal circles with the same angular velocity ω rad s^{-1} about a vertical axis through O. Show that $OQ = \frac{3l}{5}$ m. **(7 marks)**

6. The figure show a wine glass consisting of a hemispherical cup of radius r, a cylindrical solid stem of height r and a circular base of radius r. The cup has mass M and the stem has mass m. Modelling the cup as a thin, uniform hemispherical shell, the base as a uniform lamina made of the same thin material as the cup, and the stem as a uniform solid cylinder,



(a) show that the mass of the circular base is $\frac{1}{2}M$. **(1 mark)**

Given that the centre of mass of the glass is at a distance $\frac{13r}{14}$ from the base along the vertical axis of symmetry,

(b) express M in terms of m. **(6 marks)**

If the cup is now filled with liquid whose mass is $2M$,

(c) show that the position of the centre of mass rises through a distance $\frac{13r}{35}$. **(6 marks)**

(d) State an assumption that you have made about the liquid. **(1 mark)**

7. A particle of mass m kg is attached to one end of an elastic string of natural length l m and modulus of elasticity λ N. The other end of the string is attached to a fixed point O. The particle hangs in equilibrium at a point C.

(a) (i) Prove that if the particle is slightly displaced in a vertical direction, it will perform simple harmonic motion about C. **(6 marks)**

(ii) Find the period, T s, of the motion in terms of l, m and λ. **(1 mark)**

(iii) Explain the significance of the term 'slightly' as used in (i) above. **(1 mark)**

When an additional mass M is attached to the particle, it is found that the system oscillates about a point D, at a distance d below C, with period T_1 s.

(b) (i) Write down an expression for T_1 in terms of l, m, M and λ. **(2 marks)**

(ii) Hence show that $T_1^2 - T^2 = \frac{4\pi^2 d}{g}$. **(5 marks)**

