MECHANICS 3 (A) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1. $\frac{\lambda}{0\cdot3} \times 0\cdot1 = 0\cdot4 \times 9\cdot8$ $\lambda = 11\cdot76$ N   M1 A1
   Energy : $0\cdot4 \times 9\cdot8 \times (0\cdot15 + x) = \frac{11\cdot76}{2\times0\cdot3}x^2$ $0\cdot06 + 0\cdot4x = 2x^2$ M1 A1 A1
   $10x^2 - 20x - 3 = 0$ $(10x + 1)(10x - 3) = 0$ $x = 0\cdot3$ m   M1 A1   7

2. (a) Resultant force towards centre $= \frac{mv^2}{r}$ B1
   (b) Weight $= mg$, reaction R at angle θ to vertical $R\cos\theta = mg$ B1 B1 B1
   Horizontally : $R\sin\theta = \frac{mv^2}{r}$ Divide : $\tan\theta = \frac{v^2}{gr}$ M1 A1 A1   7

3. (a) $2T\sin 30° = 0\cdot2 \times 9\cdot8$ $T = 1\cdot96$ N   M1 A1
   (b) P.E. loss = K.E. gain : $0\cdot2 \times 9\cdot8 \times r(\sin 60° - \sin 30°) = \frac{1}{2} \times 0\cdot2v^2$ M1 A1 A1
   $T - 0\cdot2g\cos 30° = \frac{0\cdot2v^2}{2r}$ $T = 1\cdot96(0\cdot732 + 0\cdot866) = 3\cdot13$ N   M1 A1 A1   8

4. (a) $mg = k\frac{mx}{r^2}$ (given), so $k = gR^2$ B1
   (b) Equal periods, so $\frac{2\pi(4R)}{v_B} = \frac{2\pi(3R)}{v_A}$ $\frac{v_B}{v_A} = \frac{4}{3}$ M1 A1
   (c) $\frac{gR^2m}{9R^2} - T = \frac{mv^2}{3R}$, $\frac{gR^2m}{16R^2} + T = \frac{16mv^2}{9(4R)}$ M1 A1 A1
   $\frac{mg}{9} - T = \frac{mv^2}{3R}$, $\frac{mg}{16} + T = \frac{4mv^2}{9R}$ Solve : $T = \frac{37}{1008}mg$ M1 A1 A1 M1 A1   11

5. (a) $P : T = 0\cdot06g$ $Q : 0\cdot06g\cos\theta = 0\cdot04g$ $\cos\theta = \frac{2}{3}$ B1 M1 A1 A1
   Hence $\frac{2}{3}(l - d) = d$ $d = \frac{2}{5}l$ M1 A1
   (b) $P : T\sin\beta = 0\cdot06e\sin\beta\,\omega^2$ $T = 0\cdot06e\omega^2$ M1 A1
   $Q : T\sin\alpha = 0\cdot04(l - e)\sin\alpha\,\omega^2$ $T = 0\cdot04(l - e)\omega^2$ M1 A1
   Equate : $3e = 2(l - e)$ $e = \frac{2}{5}l$ $OQ = \frac{3}{5}l$ M1 A1 A1   13

6. (a) Cup : S.A. $= 2\pi r^2$, mass M Base : Area $= \pi r^2$, so mass $= \frac{1}{2}M$ B1
   (b) M(base) : $M\frac{3r}{2} + m\frac{r}{2} = \left(M + \frac{M}{2} + m\right)\frac{13r}{14}$ M1 M1 A1 A1
   $42M + 14m = 39M + 26m$ $3M = 12m$ $M = 4m$ M1 A1
   (c) M(base) : $\frac{7M}{4}\frac{13r}{14} + 2M\frac{13r}{8} = \left(2M + \frac{7M}{4}\right)\overline{y_1}$ $\overline{y_1} = \frac{13r}{10}$ M1 A1 M1 A1 A1
   C. of M. rises by $\frac{13r}{10} - \frac{13r}{14} = \frac{13r}{35}$ A1
   (d) Assumed liquid is a solid hemisphere   B1   14

7. (a) (i) At displ. x, $T - mg = -m\ddot{x}$ $T = \frac{\lambda}{l}(e + x)$ $mg = \frac{\lambda}{l}e$ M1 A1 B1 B1
   $\frac{\lambda e}{l} + \frac{\lambda x}{l} - mg = -m\ddot{x}$ $m\ddot{x} = -\frac{\lambda}{ml}x$ Hence SHM about C   M1 A1
   (ii) Period $= 2\pi\sqrt{\frac{lm}{\lambda}}$ (iii) String must not go slack   B1; B1
   (b) (i) m becomes $m + M$, so $T_1 = 2\pi\sqrt{\frac{(m+M)l}{\lambda}}$ M1 A1
   (ii) $T_1{}^2 - T^2 = \frac{4\pi^2}{\lambda}[l(m + M) - lm] = \frac{4\pi^2}{\lambda}lM$ M1 A1
   At D, $(m + M)g = \frac{\lambda}{l}(e + d)$, so $\frac{\lambda d}{l} = Mg$ $T_1{}^2 - T^2 = \frac{4\pi^2 d}{g}$ M1 A1 A1   15