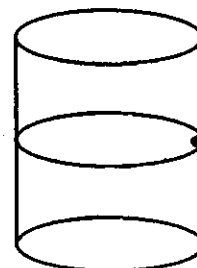


MECHANICS (A) UNIT 3**TEST PAPER 4**

Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

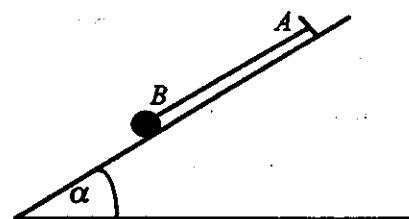
1. A motorcyclist rides in a cylindrical well of radius 5 m. He maintains a horizontal circular path at a constant speed of 10 ms^{-1} . The coefficient of friction between the wall and the wheels of the cycle is μ . Modelling the cyclist and his machine as a particle in contact with the wall, show that he will not slip downwards provided that $\mu \geq 0.49$.



(7 marks)

2. A particle P moves with simple harmonic motion in a straight line. The centre of oscillation is O . When P is at a distance 1 m from O , its speed is 8 ms^{-1} . When it is at a distance 2 m from O , its speed is 4 ms^{-1} .
- (a) Find the amplitude of the motion. (4 marks)
- (b) Show that the period of motion is $\frac{\pi}{2}$ s. (3 marks)

3. A particle of mass m kg is attached to the end B of a light elastic string AB . The string has natural length l m and modulus of elasticity λ N. The end A is attached to a fixed point on a smooth plane inclined at an angle α to the horizontal, as shown, and the particle rests in equilibrium with the length $AB = \frac{5l}{4}$ m.



- (a) Show that $\lambda = 4mg \sin \alpha$. (3 marks)
- The particle is now moved and held at rest at A with the string slack. It is then gently released so that it moves down the plane along a line of greatest slope.
- (b) Find the greatest distance from A that the particle reaches down the plane. (6 marks)

4. The acceleration $a \text{ ms}^{-2}$ of a particle P moving in a straight line away from a fixed point O is given by $a = \frac{k}{1+t}$, where t s is the time that has elapsed since P left O , and k is a constant.
- (a) By solving a suitable differential equation, find an expression for the velocity $v \text{ ms}^{-1}$ of P in terms of t , k and another constant c . (3 marks)
- Given that $v = 0$ when $t = 0$ and that $v = 4$ when $t = 2$,
- (b) show that $v \ln 3 = 4 \ln (1 + t)$. (3 marks)
- (c) Calculate the time when P has a speed of 8 ms^{-1} . (3 marks)

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5. A particle of mass m kg, at a distance x m from the centre of the Earth, experiences a force of magnitude $\frac{km}{x^2}$ N towards the centre of the Earth, where k is a constant. Given that the radius of the Earth is 6.37×10^6 m, and that a 3 kg mass experiences a force of 30 N at the surface of the Earth,

(a) calculate the value of k , stating the units of your answer. (3 marks)

The 3 kg mass falls from rest at a distance $x = 12.74 \times 10^6$ m from the centre of the Earth. Ignoring air resistance,

(b) show that it reaches the surface of the Earth with speed $7.98 \times 10^3 \text{ ms}^{-1}$. (7 marks)

In a simplified model, the particle is assumed to fall with a constant acceleration 10 ms^{-2} . According to this model it attains the same speed as in (b), $7.98 \times 10^3 \text{ ms}^{-1}$, at a distance $(12.74 - d) \times 10^6$ m from the centre of the Earth.

(c) Find the value of d . (3 marks)

6. A particle P of mass 0.4 kg hangs by a light, inextensible string of length 20 cm whose other end is attached to a fixed point O . It is given a horizontal velocity of 1.4 ms^{-1} so that it begins to move in a vertical circle. If in the ensuing motion the string makes an angle of θ with the downward vertical through O , show that

(a) θ cannot exceed 60° , (6 marks)

(b) the tension, T N, in the string is given by $T = 3.92(3 \cos \theta - 1)$. (4 marks)

If the string breaks when $\cos \theta = \frac{3}{5}$ and P is ascending,

(c) find the greatest height reached by P above the initial point of projection. (5 marks)

7. A uniform solid sphere, of radius a , is divided into two sections by a plane at a distance $\frac{a}{2}$ from the centre and parallel to a diameter.

(a) Show that the centre of gravity of the smaller cap from its plane face is $\frac{7a}{40}$. (9 marks)

This smaller cap is now placed on an inclined plane whose angle of inclination to the horizontal is θ . The plane is rough enough to prevent slipping and the cap rests with its curved surface in contact with the plane.

(b) If the maximum value of θ for which this is possible without the cap turning over is 30° , find the corresponding maximum inclination of the axis of symmetry of the cap to the vertical. (6 marks)