MECHANICS 3 (A) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. Frictional force $F = mg$; normal reaction $R = m(10^2/5) = 20m$ M1 A1 A1

   $\frac{F}{R} = \frac{g}{20} = 0{\cdot}49$ No slip if $F \le \mu R$ $\mu \ge 0{\cdot}49$ M1 A1 M1 A1 7

2. (a) $64 = n^2(a^2 - 1)$, $16 = n^2(a^2 - 4)$ Divide : $4 = \frac{a^2-1}{a^2-4}$ M1 A1

   $3a^2 = 15$ $a = \sqrt{5}$ m (= 2·24 m) M1 A1

   (b) $n^2 = \frac{64}{a^2-1} = 16$ $n = 4$ $T = \frac{2\pi}{4} = \frac{\pi}{2}$ s M1 A1 A1 7

3. (a) $T = mg \sin\alpha$ $\frac{\lambda}{l} \cdot \frac{l}{4} = mg \sin\alpha$ $\lambda = 4mg \sin\alpha$ M1 A1 A1

   (b) E.P.E. gained = grav. P.E. lost : $\frac{4mg\sin\alpha}{2l}(d - l)^2 = mg\, d \sin\alpha$ M1 A1 A1

   $2d^2 - 5ld + 2l^2 = 0$ $(2d - l)(d - 2l) = 0$ $d = 2l$ m A1 M1 A1 9

4. (a) $\frac{dv}{dt} = \frac{k}{1+t}$ $\int dv = k \int \frac{1}{1+t}\, dt$ $v = k \ln(1 + t) + c$ B1 M1 A1

   (b) $t = 0$, $v = 0$, so $c = 0$ $t = 2$, $v = 4$: $k = \frac{4}{\ln 3}$; hence result M1 A1 A1

   (c) When $v = 8$, $8 = \frac{4}{\ln 3} \ln(1 + t)$ $\ln(1 + t) = \ln 9$ $t = 8$ M1 A1 A1 9

5. (a) $30 = \frac{3k}{(6{\cdot}37\times10^6)^2}$ $k = 4{\cdot}06 \times 10^{14}$ Units N m^2 kg^{-1} or m^3 s^{-2} M1 A1 A1

   (b) $mv\frac{dv}{dx} = -\frac{km}{x^2}$ $\frac{v^2}{2} = \frac{k}{x} + c$ $v = 0$, $x = 12{\cdot}74 \times 10^6$ M1 A1 A1

   $c = -3{\cdot}19 \times 10^7$ $\frac{v^2}{2} = \frac{4{\cdot}06\times10^{14}}{x} - 3{\cdot}19 \times 10^7$ M1 A1

   When $x = 6{\cdot}37 \times 10^6$, $v = 7{\cdot}98 \times 10^3$ m s^{-1} M1 A1

   (c) $v^2 = 0 + 2 \times 10 \times d$ $v^2 = 20d$ $d = 3{\cdot}18$ M1 A1 A1 13

6. (a) Energy : $\frac{1}{2}(0{\cdot}4)(1{\cdot}4)^2 = 0{\cdot}4 \times 9{\cdot}8 \times 0{\cdot}2(1 - \cos\theta) + \frac{1}{2} \times 0{\cdot}4v^2$ M1 A1 A1

   $v^2 = 1{\cdot}96 - 3{\cdot}92(1 - \cos\theta) = 3{\cdot}92\cos\theta - 1{\cdot}96$ A1

   $v^2 \ge 0$, so $\cos\theta \ge \frac{1}{2}$ $\theta \le 60°$ M1 A1

   (b) $T - mg\cos\theta = \frac{mv^2}{r}$ $T = 0{\cdot}4 \times 9{\cdot}8 \times \cos\theta + 2(3{\cdot}92\cos\theta - 1{\cdot}96)$ B1 M1 A1

   $T = 3{\cdot}92(3\cos\theta - 1)$ A1

   (c) $u^2 = 3{\cdot}92(0{\cdot}6) - 1{\cdot}96 = 0{\cdot}392$ Energy : $\frac{1}{2}m(0{\cdot}392)^2 = mgh$ M1 A1

   $h = 0{\cdot}00784$ Greatest height = 0·08 + 0·00784 = 0·0878 m M1 A1 A1 15

7. (a) $\bar{x}\,\pi\int_{a/2}^{a}(a^2 - x^2)\,dx = \pi\int_{a/2}^{a}(a^2x - x^3)\,dx$ M1 A1 M1 A1

   $\frac{5a^3\pi\bar{x}}{24} = \frac{9a^4\pi}{64}$ $\bar{x} = \frac{9a^4}{64} \times \frac{24}{5a^3} = \frac{27a}{40}$ From O : $\frac{27a}{40} - \frac{a}{2} = \frac{7a}{40}$ M1 A1 A1 M1 A1

   (b) Reaction acts through centre O; centre of mass G on vertical B1 B1

   through point of contact S; let angle $OGS = \beta$ B1

   Sine rule in ΔOSG : $\frac{\sin\beta}{40} = \frac{\sin 30°}{27}$ $\sin\beta = \frac{20}{27}$ M1 A1

   $\beta = 132{\cdot}2°$ $\alpha = 180° - (30° + \beta) = 17{\cdot}8°$ A1 15

