

MECHANICS (A) UNIT 3**TEST PAPER 3**

Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. One end of a light inextensible string of length $2r$ m is attached to a fixed point O . A particle of mass m kg is attached to the other end Q of the string, so that it can move in a vertical plane. The string is held taut and horizontal and the particle is projected vertically downwards with a speed $\sqrt{gr} \text{ ms}^{-1}$. When the string is vertical it begins to wrap round a small, smooth peg X at a distance r m vertically below O . The particle continues to move.
 - (a) Find the speed of the particle when it reaches O , in terms of g and r . (2 marks)
 - (b) Show that, when QX is horizontal, the tension in the string is $3mg$ N. (5 marks)

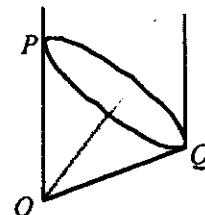
2. A particle moving along the x -axis describes simple harmonic motion about the origin O . The period of its motion is $\frac{\pi}{2}$ seconds. When it is at a distance 1 m from O , its speed is 3 ms^{-1} . Calculate
 - (a) the amplitude of its motion, (4 marks)
 - (b) the maximum acceleration of the particle, (1 mark)
 - (c) the least time that it takes to move from O to a point 0.25 m from O . (4 marks)

3. A particle P of mass m kg is attached to the mid-point of a light elastic string of natural length $8l$ m and modulus of elasticity λ N. The two ends of the string are attached to fixed points A and B on the same horizontal level, where $AB = 8l$ m. P is released from rest at the mid-point of AB .
 - (a) If P comes to instantaneous rest at a depth $3l$ m below AB , find an expression for λ in terms of m and g . (4 marks)
 - (b) Using this value of λ , show that the speed $v \text{ ms}^{-1}$ of P when it passes through the point $2l$ m below AB is given by $v^2 = 4(24\sqrt{5} - 53)gl$. (5 marks)

4. A particle P of mass 0.8 kg moves along a straight line OL and is acted on by a resistive force of magnitude R N directed *towards* the fixed point O . When the displacement of P from O is x m, $R = \frac{0.8xv^2}{1+x^2}$, where $v \text{ ms}^{-1}$ is the speed of P at that instant.
 - (a) Write down a differential equation for the motion of P . (2 marks)
Given that $v = 2$ when $x = 0$,
 - (b) find the speed with which P passes through the point A , where $OA = 1$ m. (7 marks)

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5. The diagram shows a uniform solid right circular cone of mass m kg, height h m and base radius r m suspended by two vertical strings attached to the points P and Q on the circumference of the base. The vertex O of the cone is vertically below P .

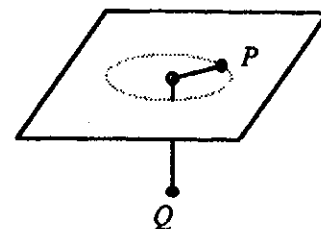


- (a) Show that the tension in the string attached at Q is $\frac{3mg}{8}$ N.
 (b) Find, in terms of m and g , the tension in the other string.

(8 marks)**(2 marks)**

6. Two identical particles P and Q are connected by a light inextensible string passing through a small smooth-edged hole in a smooth table, as shown.

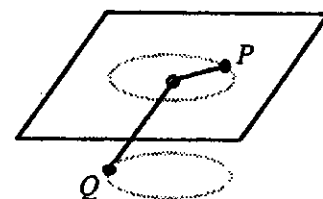
P moves on the table in a horizontal circle of radius 0.2 m and Q hangs at rest.



- (a) Calculate the number of revolutions made per minute by P .

(5 marks)

Q is now also made to move in a horizontal circle of radius 0.2 m below the table. The part of the string between Q and the table makes an angle of 45° with the vertical.



- (b) Show that the numbers of revolutions per minute made by P and Q respectively are in the ratio $2^{1/4} : 1$.

(9 marks)

7. A particle P of mass m kg is fixed to one end of a light elastic string of natural length l m and modulus of elasticity kmg N. The other end of the string is fixed to a point X on a horizontal plane. P rests at O , where $OX = l$ m, with the string just taut. It is then pulled away from X through a distance $\frac{3l}{4}$ m and released from rest. On this side of O , the plane is smooth.

- (a) Show that, as long as the string is taut, P performs simple harmonic motion. **(4 marks)**

- (b) Given that P first returns to O with speed \sqrt{gl} ms^{-1} , find the value of k . **(3 marks)**

- (c) On the other side of O the plane is rough, the coefficient of friction between P and the plane being μ . If P does not reach X in the subsequent motion, show that $\mu > \frac{1}{2}$. **(4 marks)**

- (d) If, further, $\mu = \frac{3}{4}$, show that the time which elapses after P is released and before it comes to rest is $\frac{1}{24}(9\pi + 32)\sqrt{\frac{l}{g}}$ s. **(6 marks)**