

MECHANICS 3 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1. $m \frac{dv}{dt} = -mkv \quad \int \frac{1}{v} dv = \int -k dt \quad \ln v = -kt + c$ M1 A1 M1 A1
 $\ln u = c$, so $\ln \frac{v}{u} = -kt \quad v = ue^{-kt}$ M1 A1 A1 7
2. $T \cos \theta = mg$, $T \sin \theta = m(0.4 \sin \theta)\omega^2 \quad g = 0.4\omega^2 \cos \theta$ M1 A1 M1 A1
 $\theta \leq 60^\circ$, so $\cos \theta \geq 0.5 \quad g \geq 0.2\omega^2 \quad \omega^2 \leq 49 \quad \omega \leq 7$ B1 M1 A1 7
3. (a) $mg = \frac{mge}{2(0.5)} \quad e = 0.5 \times 2 = 1 \text{ m} \quad OP = 1.5 \text{ m}$ M1 A1 A1
 (b) P.E. = $\frac{mg(l^2)}{2(2)(0.5)} = \frac{mg}{2} \text{ J}$ (c) Work done = $mg \times 1.5 = \frac{3mg}{2} \text{ J}$ M1 A1 M1 A1
 (d) Grav. P.E. lost > elastic P.E. gained, because the weight does work in moving the supportive force in (c) B1 8
4. Symmetric, so tensions in strings are equal $2T \cos \theta = mg$ B1 M1 A1
 $AP \sin \theta = l$, so $AP = \frac{l}{\sin \theta} \quad T = \frac{3mg}{l} \left(\frac{l}{\sin \theta} - l \right)$ M1 A1
 Hence $2 \times 3mg \left(\frac{l}{\sin \theta} - l \right) \cos \theta = mg \quad 6(\cot \theta - \cos \theta) = 1$, etc M1 A1 A1 8
5. (a) P.E. gained = K.E. lost: $mgr(1 - \cos \theta) = \frac{1}{2} m (3gr) - \frac{1}{2} mv^2$ M1 M1 A1
 $v^2 = 3gr - 2gr + 2gr \cos \theta = gr(1 + 2 \cos \theta)$ M1 A1
 (b) $R - mg \cos \theta = \frac{mv^2}{r}$ M1 A1
 $R = \frac{m}{r} gr(1 + 2 \cos \theta) + mg \cos \theta \quad R = mg(1 + 3 \cos \theta)$ M1 A1
 (c) (i) When $v = 0$, $\cos \theta = -\frac{1}{2}$ (ii) When $R = 0$, $\cos \theta = -\frac{1}{3}$ B1 B1
 (d) $h_1 = r + \frac{1}{2}r = \frac{3r}{2} \quad h_2 = r + \frac{1}{3}r = \frac{4r}{3} \quad \text{Ratio} = \frac{3r}{2} + \frac{4r}{3} = \frac{9}{6} + \frac{8}{6} = \frac{17}{6}$ M1 A1 A1 14
6. (a) $mg = \frac{\lambda}{0.8} \times 0.7 = 0.5 \times 9.8 \quad \lambda = 4.9 \times \frac{0.8}{0.7} = 5.6 \text{ N}$ M1 A1 A1
 (b) $(0.5 \times 9.8) - \frac{5.6}{0.8} (0.7 + x) = 0.5x \quad 4.9 - 4.9 - 7x = 0.5x$ M1 A1
 $x = -14x$, of form $x = n^2x$ with $n^2 = 14$, so simple harmonic A1 A1
 Period = $2\pi/\sqrt{14} = 1.68 \text{ s}$ A1
 (c) Maximum speed = $an = 0.5 \sqrt{14} = 1.87 \text{ ms}^{-1}$ B1 M1 A1
 (d) $x = 0.5 \cos nt \quad 0.25 = 0.5 \cos nt \quad \cos nt = 0.5$ B1 M1
 $nt = \frac{\pi}{3} \quad t = 0.28 \text{ s}$ A1 A1 15
7. (a) $x^2 + y^2 = r^2 \quad \bar{x} \int_0^r \pi y^2 dx = \int_0^r \pi xy^2 dx$ B1 M1 A1
 $\pi \bar{x} \int_0^r r^2 - x^2 dx = \pi \int_0^r r^2 x - x^3 dx \quad \frac{2r^2}{3} \bar{x} = \frac{r^4}{4} \quad \bar{x} = \frac{3r}{8}$ M1 A1 A1 A1
 (b) Forces shown: weight, normal reaction (thro' O), friction at plane B1 B1 B1
 (c) Resolve // plane: $F = W \sin \alpha \quad M(O): Fr = W \frac{3r}{8} \cos \alpha$ B1 M1 A1
 $Wr \sin \theta = \frac{3r}{8} W \cos \alpha \quad \cos \alpha = \frac{8}{3} \times \frac{3}{10} = \frac{4}{5}$ M1 A1 A1 16