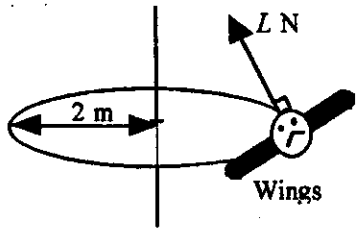
MECHANICS (A) UNIT 3 **TEST PAPER 1**

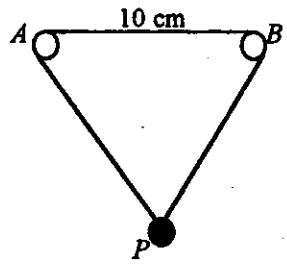
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A bird of mass 0·5 kg, flying around a vertical feeding post at a constant speed of 6 ms^{-1}, banks its wings to move in a horizontal circle of radius 2 m. The aerodynamic lift L newtons is perpendicular to the bird's wings, as shown.



Modelling the bird as a particle, find, to the nearest degree, the angle that its wings make with the vertical. **(7 marks)**

2. A thin elastic string, of modulus λ N and natural length 20 cm, passes round two small, smooth pegs A and B on the same horizontal level to form a closed loop. $AB = 10$ cm. The ends of the string are attached to a weight P of mass 0·7 kg. When P rests in equilibrium, APB forms an equilateral triangle.



(a) Find the value of λ. **(6 marks)**

(b) State one assumption that you have made about the weight P, explaining how you have used this assumption in your solution. **(1 mark)**

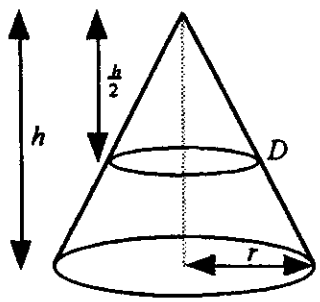
3. A particle P of mass 0·5 kg moves along a straight line. When P is at a distance x m from a fixed point O on the line, the force acting on it is directed towards O and has magnitude $\frac{8}{x^2}$ N. When $x = 2$, the speed of P is 4 ms^{-1}.

Find the speed of P when it is 0·5 m from O. **(8 marks)**

4. A particle P of mass m kg is attached to one end of a light elastic string of natural length l m and modulus of elasticity λ N. The other end of the string is attached to a fixed point O. P is released from rest at O and falls vertically downwards under gravity. The greatest distance below O reached by P is $2l$ m.

(a) Show that $\lambda = 4mg$. **(3 marks)**

(b) Find, in terms of l and g, the speed with which P passes through the point A, where $OA = \frac{5l}{4}$ m. **(6 marks)**

5. 

A uniform solid right circular cone has height h and base radius r. The top part of the cone is removed by cutting through the cone parallel to the base at a height $\frac{h}{2}$.

(a) Show that the centre of mass of the remaining solid is at a height $\frac{11h}{56}$ above the base, along its axis of symmetry. **(7 marks)**

The remaining part of the solid is suspended from the point D on the circumference of its smaller circular face, and the axis of symmetry then makes an angle α with the vertical, where $\tan\alpha = \frac{1}{2}$.

(b) Find the value of the ratio $h : r$. **(5 marks)**

6. A light elastic string, of natural length l m and modulus of elasticity $\frac{mg}{2}$ newtons, has one end fastened to a fixed point O. A particle P, of mass m kg, is attached to the other end of the string. P hangs in equilibrium at the point E, vertically below O, where $OE = (l + e)$ m

(a) Find the numerical value of the ratio $e : l$. **(2 marks)**

P is now pulled down a further distance $\frac{3l}{2}$ m from E and is released from rest.

In the subsequent motion, the string remains taut. At time t s after being released, P is at a distance x m below E.

(b) Write down a differential equation for the motion of P and show that the motion is simple harmonic. **(4 marks)**

(c) Write down the period of the motion. **(2 marks)**

(d) Find the speed with which P first passes through E again. **(2 marks)**

(e) Show that the time taken by P after it is released to reach the point A above E, where $AE = \frac{3l}{4}$ m, is $\frac{2\pi}{3}\sqrt{\frac{2l}{g}}$ s. **(5 marks)**

7. A particle P is attached to one end of a light inextensible string of length l m. The other end of the string is attached to a fixed point O. When P is hanging at rest vertically below O, it is given a horizontal speed u ms^{-1} and starts to move in a vertical circle.

Given that the string becomes slack when it makes an angle of 120° with the downward vertical through O,

(a) show that $u^2 = \frac{7gl}{2}$. **(10 marks)**

(b) Find, in terms of l, the greatest height above O reached by P in the subsequent motion. **(7 marks)**

