AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 3

Wednesday 8 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 At a firing range, a man holds a gun and fires a bullet horizontally. The bullet is fired with a horizontal velocity of $400\,\mathrm{m\,s^{-1}}$. The mass of the gun is $1.5\,\mathrm{kg}$ and the mass of the bullet is 30 grams.

(a) Find the speed of recoil of the gun.

[2 marks]

(b) Find the magnitude of the impulse exerted by the man on the gun in bringing the gun to rest after the bullet is fired.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	**Answer space for question 1**

2 A lunar mapping satellite of mass m_1 measured in kg is in an elliptic orbit around the moon, which has mass m_2 measured in kg. The effective potential, E, of the satellite is given by

$$E = \frac{K^2}{2m_1r^2} - \frac{Gm_1m_2}{r}$$

where r measured in metres is the distance of the satellite from the moon, $G\ \text{Nm}^2\text{kg}^{-2}$ is the universal gravitational constant, and K is the angular momentum of the satellite.

By using dimensional analysis, find the dimensions of:

(a) E,

[3 marks]

(b) K.

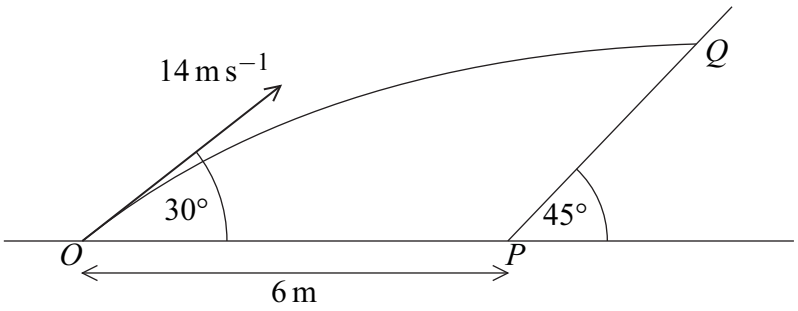
[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

3 A ball is projected from a point O on horizontal ground with speed $14\,\text{m s}^{-1}$ at an angle of elevation $30°$ above the horizontal. The ball travels in a vertical plane through the point O and hits a point Q on a plane which is inclined at $45°$ to the horizontal. The point O is 6 metres from P, the foot of the inclined plane, as shown in the diagram. The points O, P and Q lie in the same vertical plane. The line PQ is a line of greatest slope of the inclined plane.



(a) During its flight, the horizontal and upward vertical distances of the ball from O are x metres and y metres respectively.

Show that x and y satisfy the equation

$$y = x\frac{\sqrt{3}}{3} - \frac{x^2}{30}$$

Use $\cos 30° = \frac{\sqrt{3}}{2}$ and $\tan 30° = \frac{\sqrt{3}}{3}$.

[5 marks]

(b) Find the distance PQ.

[7 marks]

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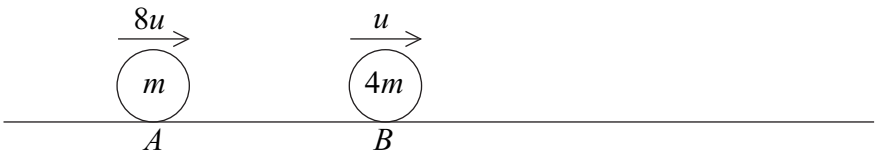
4 A smooth uniform sphere A, of mass m, is moving with velocity $8u$ in a straight line on a smooth horizontal table. A smooth uniform sphere B, of mass $4m$, has the same radius as A and is moving on the table with velocity u.



The sphere A collides directly with the sphere B.

The coefficient of restitution between A and B is e.

(a) (i) Find, in terms of u and e, the velocities of A and B immediately after the collision.

[6 marks]

(ii) The direction of motion of A is reversed by the collision. Show that $e > a$, where a is a constant to be determined.

[2 marks]

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{2}{3}$.

The sphere B collides with A again after rebounding from the wall.

Show that $e < b$, where b is a constant to be determined.

[3 marks]

(c) Given that $e = \frac{4}{7}$, find, in terms of m and u, the magnitude of the impulse exerted on B by the wall.

[3 marks]

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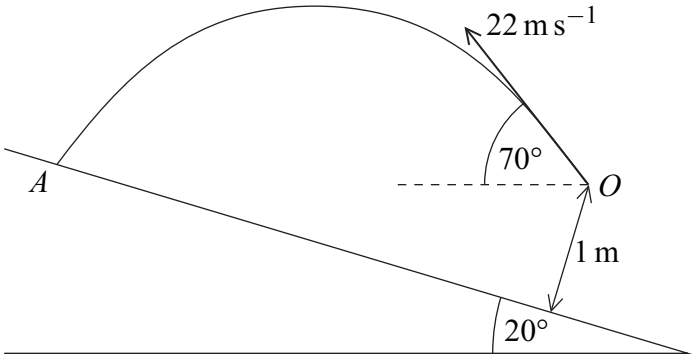
5 A ball is projected from a point O above a smooth plane which is inclined at an angle of $20°$ to the horizontal. The point O is at a perpendicular distance of $1\,\text{m}$ from the inclined plane. The ball is projected with velocity $22\,\text{m s}^{-1}$ at an angle of $70°$ above the **horizontal**. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane for the first time at a point A.



(a) (i) Find the time taken by the ball to travel from O to A.

[4 marks]

(ii) Find the components of the velocity of the ball, parallel and perpendicular to the inclined plane, as it strikes the plane at A.

[4 marks]

(b) After striking A, the ball rebounds and strikes the plane for a second time at a point further up than A.

The coefficient of restitution between the ball and the inclined plane is e.

Show that $e < k$, where k is a constant to be determined.

[4 marks]

QUESTION PART REFERENCE

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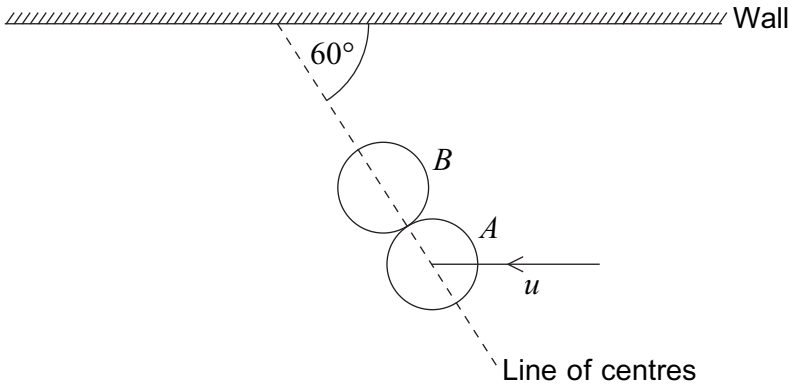
6 In this question use $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$.

A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table. Just before the collision, the direction of motion of A is parallel to a fixed smooth vertical wall. At the instant of collision, the line of centres of A and B makes an angle of $60°$ with the wall, as shown in the diagram.



The coefficient of restitution between A and B is e.

(a) Show that the speed of B immediately after the collision is $\frac{1}{4}u(1+e)$ and find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision.

[7 marks]

(b) Subsequently, B collides with the wall. After colliding with the wall, the direction of motion of B is parallel to the direction of motion of A after its collision with B.

Show that the coefficient of restitution between B and the wall is $\frac{1+e}{7-e}$.

[7 marks]

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7 A quad-bike, a truck and a car are moving on a large, open, horizontal surface in a desert plain. Relative to the quad-bike, which is travelling due west at its maximum speed of $10\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $340°$. Relative to the car, which is travelling due east at a speed of $15\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $300°$.

(a) Show that the speed of the truck is approximately $24.7\,\mathrm{m\,s^{-1}}$ and that it is moving on a bearing of $318°$, correct to the nearest degree.

[8 marks]

(b) At the instant when the truck is at a distance of 400 metres from the quad-bike, the bearing of the truck from the quad-bike is $060°$. The truck continues to move with the same velocity as in part **(a)**. The quad-bike continues to move at a speed of $10\,\mathrm{m\,s^{-1}}$.

Find the bearing, to the nearest degree, on which the quad-bike should travel in order to approach the truck as closely as possible.

[5 marks]

QUESTION PART REFERENCE

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END OF QUESTIONS