Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MM03

Unit Mechanics 3

Friday 6 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A tennis ball is projected from a point O with a velocity of $(4\sqrt{3}\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$, where **i** and **j** are horizontal and vertical unit vectors respectively. The ball travels in a vertical plane through O which is $30\,\mathrm{cm}$ above the horizontal surface of a tennis court. During its flight, the horizontal and upward vertical distances of the ball from O are x metres and y metres respectively.

Model the ball as a particle.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$$

[4 marks]

(b) The ball hits a vertical net at a point A. The net is at a horizontal distance of $4\,\mathrm{m}$ from O.

Determine the height of the point A, above the surface of the tennis court. Give your answer to the nearest centimetre.

[2 marks]

(c) State a modelling assumption, other than the ball being a particle, that you need to make to answer this question.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

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2 A rod, of length $x\,\text{m}$ and moment of inertia $I\,\text{kg}\,\text{m}^2$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through one end.

When the rod is hanging at rest, its lower end receives an impulse of magnitude $J\,\text{Ns}$, which is just sufficient for the rod to complete full revolutions.

It is thought that there is a relationship between J, x, I, the acceleration due to gravity $g\,\text{m}\,\text{s}^{-2}$ and a dimensionless constant k, such that

$$J = kx^{\alpha}I^{\beta}g^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ for which this relationship is dimensionally consistent.

[6 marks]

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3 A particle of mass $0.5\,\text{kg}$ is moving in a straight line on a smooth horizontal surface.

The particle is then acted on by a horizontal force for 3 seconds. This force acts in the direction of motion of the particle and at time t seconds has magnitude $(3t + 1)\ \text{N}$.

When $t = 0$, the velocity of the particle is $4\,\text{m s}^{-1}$.

(a) Find the magnitude of the impulse of the force on the particle between the times $t = 0$ and $t = 3$.

[3 marks]

(b) Hence find the velocity of the particle when $t = 3$.

[2 marks]

(c) Find the value of t when the velocity of the particle is $20\,\text{m s}^{-1}$.

[4 marks]

QUESTION PART REFERENCE

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4 Two boats, A and B, are moving on straight courses with constant speeds. At noon, A and B have position vectors $(\mathbf{i} + 2\mathbf{j})\,\text{km}$ and $(-\mathbf{i} + \mathbf{j})\,\text{km}$ respectively relative to a lighthouse. Thirty minutes later, the position vectors of A and B are $(-\mathbf{i} + 3\mathbf{j})\,\text{km}$ and $(2\mathbf{i} - \mathbf{j})\,\text{km}$ respectively relative to the lighthouse.

(a) Find the velocity of A relative to B in the form $(m\mathbf{i} + n\mathbf{j})\,\text{km h}^{-1}$, where m and n are integers.

[4 marks]

(b) The position vector of A relative to B at time t hours after noon is $\mathbf{r}\,\text{km}$.

Show that

$$\mathbf{r} = (2 - 10t)\mathbf{i} + (1 + 6t)\mathbf{j}$$

[3 marks]

(c) Determine the value of t when A and B are closest together.

[5 marks]

(d) Find the shortest distance between A and B.

[2 marks]

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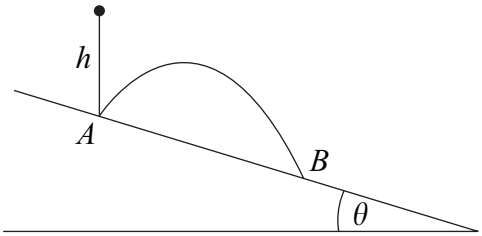
5 A small smooth ball is dropped from a height of h above a point A on a fixed smooth plane inclined at an angle θ to the horizontal. The ball falls vertically and collides with the plane at the point A. The ball rebounds and strikes the plane again at a point B, as shown in the diagram. The points A and B lie on a line of greatest slope of the inclined plane.



(a) Explain whether or not the component of the velocity of the ball parallel to the plane is changed by the collision.

[2 marks]

(b) The coefficient of restitution between the ball and the plane is e.

Find, in terms of h, θ, e and g, the components of the velocity of the ball parallel to and perpendicular to the plane immediately after the collision.

[3 marks]

(c) Show that the distance AB is given by

$$4he(e+1)\sin\theta$$

[7 marks]

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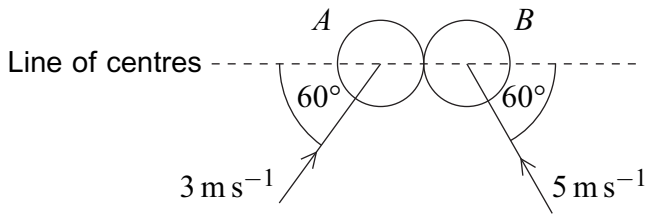
6 Two smooth spheres, A and B, have equal radii and masses $2\,\text{kg}$ and $4\,\text{kg}$ respectively.

The spheres are moving on a smooth horizontal surface and collide. As they collide, A has velocity $3\,\text{m s}^{-1}$ at an angle of $60°$ to the line of centres of the spheres, and B has velocity $5\,\text{m s}^{-1}$ at an angle of $60°$ to the line of centres, as shown in the diagram.



Just after the collision, B moves in a direction perpendicular to the line of centres.

(a) Find the speed of A immediately after the collision.

[6 marks]

(b) Find the acute angle, correct to the nearest degree, between the velocity of A and the line of centres immediately after the collision.

[2 marks]

(c) Find the coefficient of restitution between the spheres.

[2 marks]

(d) Find the magnitude of the impulse exerted on B during the collision.

[2 marks]

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7 Two small smooth spheres, A and B, are the same size and have masses $2m$ and m respectively. Initially, the spheres are at rest on a smooth horizontal surface. The sphere A receives an impulse of magnitude J and moves with speed $2u$ directly towards B.

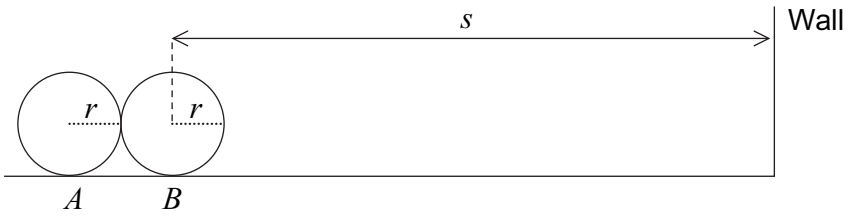
(a) Find J in terms of m and u.

[2 marks]

(b) The sphere A collides directly with B. The coefficient of restitution between A and B is $\frac{2}{3}$. Find, in terms of u, the speeds of A and B immediately after the collision.

[5 marks]

(c) At the instant of collision, the centre of B is at a distance s from a fixed smooth vertical wall which is at right angles to the direction of motion of A and B, as shown in the diagram.

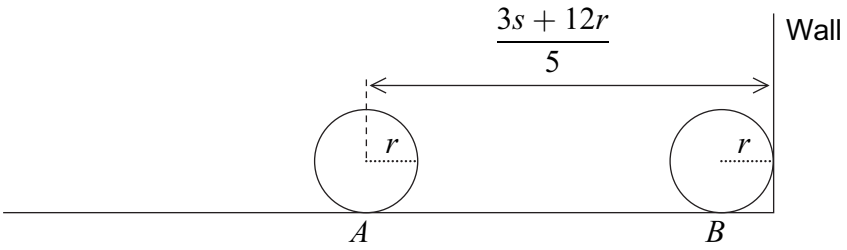


Subsequently, B collides with the wall. The radius of each sphere is r.

Show that the distance of the centre of A from the wall at the instant that B hits the wall is $\frac{3s + 12r}{5}$.

[4 marks]

(d) The diagram below shows the positions of A and B when B hits the wall.



The sphere B collides with A again after rebounding from the wall. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

Find the distance of the **centre of B** from the wall at the instant when A and B collide again.

[4 marks]

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END OF QUESTIONS

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