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General Certificate of Education (A-level)
June 2013

Mathematics MM03

(Specification 6360)

Mechanics 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

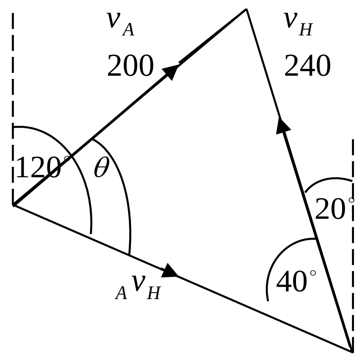
Q	Solution	Marks	Total	Comments
1	Use of Impulse-momentum principle	M1		$\int_{(0)}^{(T)} (3t+1)\,dt = \pm 2(5) \pm 2(1)$
	$\int_{(0)}^{(T)} (3t+1)\,dt = 2(5) - 2(1)$	A1		Condone sign error for M1 A1 for all correct
	$\left[\frac{3}{2}t^2 + t\right]_{(0)}^{(T)} = (8)$	A1		Correct integration, PI by the correct quadratic
	$3T^2 + 2T - 16 = 0$	A1		Correct use of correct limits and rearrangement
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{4 - 4(3)(-16)}}{2(3)}$	m1		Solution of their quadratic, **correct** attempt needed
	($T = -\frac{8}{3}$ unacceptable, not in the interval $0 \le t \le 3$)			
	$\underline{T = 2}$ √	A1	6	
	Total		**6**	
2	$[P]$ = $MLT^{-2}.L.T^{-1}$ = ML^2T^{-3}	B1		For correct unsimplified dimensions of quantities
	$[mgv\sin\theta]$ = $M.LT^{-2}.LT^{-1}$ = ML^2T^{-3}	B1		
	$[Rv]$ = $MLT^{-2}.LT^{-1}$ = ML^2T^{-3}	B1		
	$\left[\frac{1}{2}mv^3\frac{\sin\theta}{h}\right]$ = $M.L^3T^{-3}.L^{-1}$ = ML^2T^{-3}	B1		
		B1		All simplifications correct
	The formula is dimensionally consistent	E1		Dependent on the last B1
	Total		**6**	

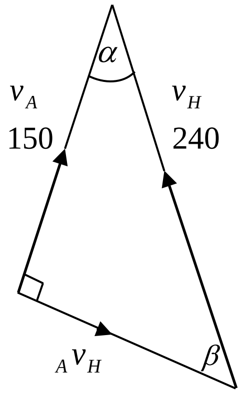
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\theta$	M1		
	$t = \dfrac{x}{u\cos\theta}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	M1		Condone $+g$ for M1
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	A1		
	$y = -\dfrac{1}{2}g(\dfrac{x}{u\cos\theta})^2 + u(\dfrac{x}{u\cos\theta})\sin\theta$	m1		Elimination of *t*, condone $+g$ for m1
	$y = -\dfrac{gx^2}{2u^2\cos^2\theta} + \dfrac{x\sin\theta}{\cos\theta}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\theta) + x\tan\theta$	A1	6	OE in terms of x, u, g, $\tan\theta$
(b)(i)	$0.5 = -\dfrac{9.8(5)^2}{2(8)^2}(1+\tan^2\theta) + 5\tan\theta$	M1		Correctly substituting for x, y, u and g into their equation of trajectory
		A1		All correct, condone decimal approximation.
	$245\tan^2\theta - 640\tan\theta + 309 = 0$	A1		OE exact quadratic in $\tan\theta$
	$\tan\theta = \dfrac{640 \pm \sqrt{(-640)^2 - 4(245)(309)}}{2(245)}$	m1		PI by the values of $\tan\theta$
	$\tan\theta = 1.973(004)$, $0.6392(41)$			
	$\theta = 63.12^\circ$, 32.58°			
	$\theta = 63.1^\circ$, 32.6°	A1	5	AG Must see the above or more accurate values
(ii)	$\dot{y} = -9.8(\dfrac{5}{8\cos 63.1^\circ}) + 8\sin 63.1^\circ$ OE	M1		Condone +9.8 for M1.
	$(\dot{y} = -6.4035)$			
	$\dot{x} = 8\cos 63.1^\circ$	M1		
	$(\dot{x} = 3.6195)$			
	$\tan^{-1}\dfrac{6.4(035)}{3.6(195)}$ $(= 61^\circ)$ OE	m1		PI by correct angle in a statement
	Direction : 61° to the horizontal or 29° to the vertical	A1	4	Have to see “horizontal” or “vertical” or diagram

Q	Solution	Marks	Total	Comments
3(b)(ii)	**Alternative:**			
	$\frac{dy}{dx} = -\frac{2gx}{2u^2}(1+\tan^2\theta)+\tan\theta$	(M1)		
	$= -\frac{2\times 9.8\times 5}{2\times 8^2}(1+\tan^2 63.1^\circ)+\tan 63.1^\circ$ $=-1.7692$	(A1)		
	$\tan^{-1}(-1.7692) = -60.52368^\circ$	(m1)		
	Direction: 61° to the horizontal or 29° to the vertical	(A1)		
(c)	The ball is a particle, or No air resistance, or The ball does not spin	B1	1	
	Total		**16**	
4(a)	$m(4u)+3m(2u) = mv_A + 3mv_B$	M1 A1		M1 for four correct momentum terms with any signs. A1for all correct
	$\frac{v_B - v_A}{4u-2u} = e$	M1 A1		M1 for correct terms for any signs, A1 for all correct.
	$\begin{pmatrix} v_A + 3v_B = 10u \\ v_B - v_A = 2ue \\ 4v_B = 2ue + 10u \end{pmatrix}$			
	$v_B = \frac{u}{2}(e+5)$	A1		OE, simplified
	$\left(v_A = \frac{u}{2}(e+5) - 2ue\right)$			
	$v_A = \frac{u}{2}(-3e+5)$	A1	6	OE, simplified
(b)	$e \le 1 \;\Rightarrow\; v_B \le \frac{u}{2}(1+5)$	M1		Use of $e \le 1$ (OE) needed
	$\Rightarrow\; v_B \le 3u$	A1	2	FT their v_B
(c)	$(I =)\ 3m\,.\,\frac{u}{2}(\frac{2}{3}+5) - 3m\,.\,2u$	M1		M1 for a difference of two momentums FT their velocity from part (a)
		A1F		A1F for their '*Final B – Initial B*'
	$= \frac{5mu}{2}$ or $2.5mu$	A1	3	
	Total		**11**	

Q	Solution	Marks	Total	Comments
5(a)	$\perp$ to plane $y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	M1		For M1, $\sin\alpha$ and $\cos\theta$ must be in the correct terms but accept $+g$.
	$y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	A1		
	$uT\sin\alpha - \frac{1}{2}gT^2\cos\theta = 0$	m1		Accept $+g$ for m1.
	$u = \dfrac{Tg\cos\theta}{2\sin\alpha}$	A1	4	OE
(b)	t or $T = \dfrac{2u\sin\alpha}{g\cos\theta}$	B1		
	$\parallel$ to plane $x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	M1		For M1, $\cos\alpha$ and $\sin\theta$ must be in the correct terms but accept $-g$.
	$x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	A1		
	$(\overline{OP} =)\ u(\dfrac{2u\sin\alpha}{g\cos\theta})\cos\alpha + \frac{1}{2}g(\dfrac{2u\sin\alpha}{g\cos\theta})^2\sin\theta$	m1		Elimination of t substituting their expression into their equation for x.
	$\left(= \dfrac{2u^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \dfrac{2u^2\sin^2\alpha\sin\theta}{g\cos^2\theta}\right)$			
	$= \dfrac{2u^2\sin\alpha(\cos\alpha\cos\theta + \sin\alpha\sin\theta)}{g\cos^2\theta}$	m1		OE single correct fraction in factorised form
	$= \dfrac{2u^2\sin\alpha\cos(\alpha-\theta)}{g\cos^2\theta}$	A1	6	AG Sight of the above line needed
	Total		**10**	

Q	Solution	Marks	Total	Comments
6	(Let $v_B = a\mathbf{i} - b\mathbf{j}$)			
	$\frac{a}{b} = \frac{3}{2}$	M1		Allow sign error
	$\frac{a}{b} = \frac{3}{2}$	A1		OE
	(Squares are smooth ⇒ j component ⇒) $b = 3$	B1		
	$a = \frac{9}{2}$	A1	4	AG
	$\left(v_B = \frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)$			
(b)	(C.L.M. along the line of centres:)			
	$4(4) - 2(2) = 4(v_A) + 2(\frac{9}{2})$	M1		OE, No sign errors
	$v_A = \frac{3}{4}$	A1		
	(Restitution along the line of centres:)			
	$e = \dfrac{-\frac{3}{4} + \frac{9}{2}}{4 + 2}$ OE	M1 A1		M1 for correct terms, A0 for sign error
	$e = \frac{5}{8}$	A1	5	
(c)	(I = Change in momentum of B along the line of centres)			
	$= 2\,(\frac{9}{2}\mathbf{i}) - 2\,(-2\mathbf{i})$	M1		Allow sign error and missing **i**
	$= 13\mathbf{i}$	A1		A0 for magnitude or $-13\mathbf{i}$
	Ns or kg m s^{-1}	B1	3	
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)(i)		B1		Correct diagram with or without arrows.
		B1		40° marked correctly, PI by correct method.
	$\frac{\sin\theta}{240}=\frac{\sin 40}{200}$	M1		Correct sine rule allowing their angle opposite 200 in their diagram.
	$\theta = 50.47483^\circ$ or 50.5°	A1		AWRT 50.5°, PI by correct bearing
	Bearing of $v_A = 069.5^\circ$	A1	5	Allow 69.5°
(a)(ii)	$\frac{{}_Av_H}{\sin(180^\circ-40^\circ-50.5^\circ)}=$ $\frac{200}{\sin 40^\circ}$ or $\frac{240}{\sin 50.5^\circ}$	M1		Allow using their angle from part (a)(i).
	${}_Av_H = 311.13408$ or 311	A1F		FT their angle from part (a)(i)
	Time $=\frac{20}{311.13408}$	M1		PI by correct answer. Allow their ${}_Av_H$.
	$(=0.0642809 \text{ hours})$ = 3.86 min	A1F	4	3sf required

Q	Solution	Marks	Total	Comments
7(b)		M1 A1		Right-angled triangle with 240 and 150 marked. Correct orientation
	$\cos\alpha = \frac{150}{240}$ or $\sin\beta = \frac{150}{240}$	M1		
	$\alpha = 51.3^\circ$ or $\beta = 38.7^\circ$	A1		PI by correct bearing
	Bearing: 031.3°	A1	5	Allow 31.3°
	Total		**14**	
	TOTAL		**75**	