Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM03

Unit Mechanics 3

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A ball of mass 0.2 kg is hit directly by a bat. Just before the impact, the ball is travelling horizontally with speed $18\,\mathrm{m\,s^{-1}}$. Just after the impact, the ball is travelling horizontally with speed $32\,\mathrm{m\,s^{-1}}$ in the opposite direction.

(a) Find the magnitude of the impulse exerted on the ball. *(2 marks)*

(b) At time t seconds after the ball first comes into contact with the bat, the force exerted by the bat on the ball is $k(0.9t - 10t^2)$ newtons, where k is a constant and $0 \leqslant t \leqslant 0.09$. The bat stays in contact with the ball for 0.09 seconds.

Find the value of k. *(4 marks)*

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2 The time, t, for a single vibration of a piece of taut string is believed to depend on

the length of the taut string, l,
the tension in the string, F,
the mass per unit length of the string, q, and
a dimensionless constant, k,

such that

$$t = kl^{\alpha}F^{\beta}q^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

QUESTION PART REFERENCE

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3 **(In this question, use $g = 10\,\text{m s}^{-2}$.)**

A golf ball is hit from a point O on a horizontal golf course with a velocity of $40\,\text{m s}^{-1}$ at an angle of elevation θ. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^2 \tan^2 \theta - 320x \tan \theta + (x^2 + 320y) = 0 \qquad \textit{(6 marks)}$$

(b) (i) The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of θ. *(5 marks)*

(ii) Which value of θ gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of θ. *(2 marks)*

QUESTION PART REFERENCE

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4 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

A helicopter, A, is travelling in the direction of the vector $-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ with constant speed $140\,\text{km h}^{-1}$. Another helicopter, B, is travelling in the direction of the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ with constant speed $60\,\text{km h}^{-1}$.

(a) Find the velocity of A relative to B. *(5 marks)*

(b) Initially, the position vectors of A and B are $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\,\text{km}$ and $(-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})\,\text{km}$ respectively, relative to a fixed origin.

Write down the position vector of A relative to B, t hours after they leave their initial positions. *(2 marks)*

(c) Find the distance between A and B when they are closest together. *(8 marks)*

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5 A ball is dropped from a height of 2.5 m above a horizontal floor. The ball bounces repeatedly on the floor.

(a) Find the speed of the ball when it first hits the floor. *(2 marks)*

(b) The coefficient of restitution between the ball and the floor is e.

(i) Show that the time taken between the first contact of the ball with the floor and the second contact of the ball with the floor is $\frac{10e}{7}$ seconds. *(3 marks)*

(ii) Find, in terms of e, the time taken between the second contact and the third contact of the ball with the floor. *(1 mark)*

(c) Find, in terms of e, the total vertical distance travelled by the ball from when it is dropped until its third contact with the floor. *(5 marks)*

(d) State a modelling assumption for answering this question, other than the ball being a particle. *(1 mark)*

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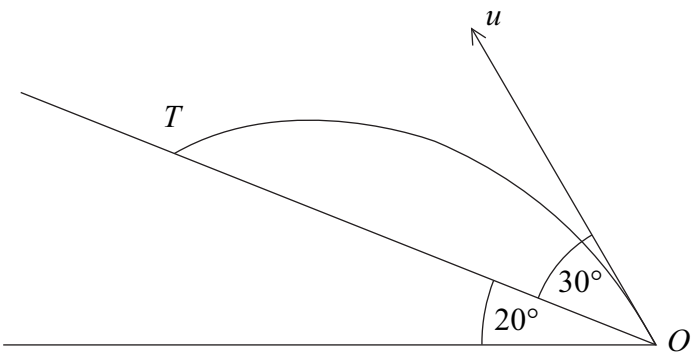
6 A projectile is fired from a point O on a plane which is inclined at an angle of 20° to the horizontal. The projectile is fired up the plane with velocity $u\,\mathrm{m\,s^{-1}}$ at an angle of 30° to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

The projectile hits a target T on the inclined plane.



(a) Given that $OT = 200\,\mathrm{m}$, determine the value of u. *(7 marks)*

(b) Find the greatest perpendicular distance of the projectile from the inclined plane. *(4 marks)*

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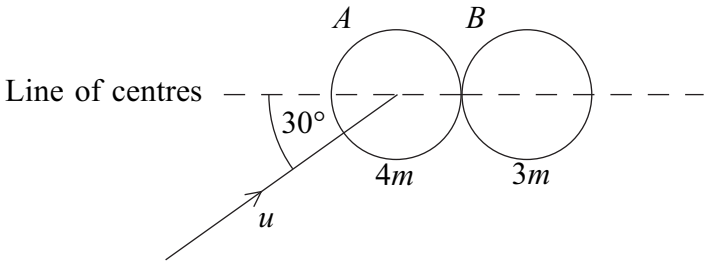
7 Two smooth spheres, A and B, have equal radii and masses $4m$ and $3m$ respectively. The sphere A is moving on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface.

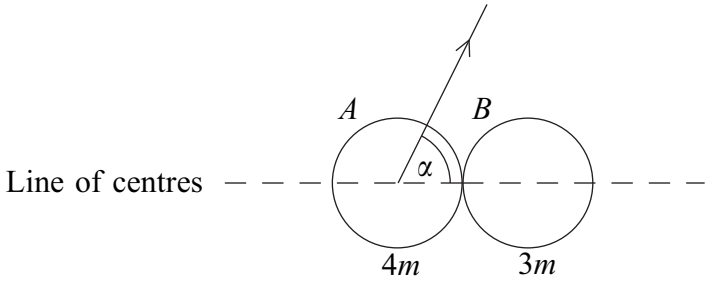
Just before the collision, A is moving with speed u at an angle of 30° to the line of centres, as shown in the diagram below.

Before collision



Immediately after the collision, the direction of motion of A makes an angle α with the line of centres, as shown in the diagram below.

After collision



The coefficient of restitution between the spheres is $\frac{5}{9}$.

(a) Find the value of α. *(10 marks)*

(b) Find, in terms of m and u, the magnitude of the impulse exerted on B during the collision. *(3 marks)*

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END OF QUESTIONS

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