Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics **MM03**

(Specification 6360)

Mechanics 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1 (a)	I = 0.2(32) + 0.2(18)	M1		
	I = 10 Ns	A1	2	Condone +10
(b)	$\int_0^{0.09} k(0.9t\text{-}10t^2)\mathrm{d}t = 10$	M1		Condone limits
	$k\left[0.45t^2 - \frac{10}{3}t^3\right]_0^{0.09} = 10$	A1F		Condone limits
	$1.215\times10^{-3}k = 10$	m1		For substituting 0.09
	$k = 8230$	A1F	4	
			6	
2	$\mathrm{T}^1 = \mathrm{L}^\alpha(\mathrm{MLT}^{-2})^\beta(\mathrm{ML}^{-1})^\gamma$	M1 A1		
	$\alpha+\beta-\gamma=0$ $\beta+\gamma=0$ $-2\beta=1$	m1		Getting three equations
	$\beta=-\frac{1}{2}$	m1		
	$\gamma=\frac{1}{2}$ $\alpha=1$	A1F	5	Solution
			5	

Q	Solution	Marks	Total	Comments
3 (a)	$x = 40\cos\theta.t$	M1		
	$y = -\frac{1}{2}(10)t^2 + 40\sin\theta.t$	M1 A1		
	$y = -\frac{1}{2}(10)(\frac{x}{40\cos\theta})^2 + 40\sin\theta.(\frac{x}{40\cos\theta})$	m1		Dependent on both M1s
	$y = -\frac{x^2}{320\cos^2\theta} + x\tan\theta$			
	$320y = -x^2(1+\tan^2\theta) + 320x\tan\theta$	m1		
	$x^2\tan^2\theta - 320x\tan\theta + (x^2 + 320y) = 0$	A1	6	Answer Given (Condone missing brackets)
(b)(i)	$150^2\tan^2\theta - 320(150)\tan\theta + (150^2 + 320\times 8) = 0$	M1		
	$1125\tan^2\theta - 2400\tan\theta + 1253 = 0$	A1		Correct quadratic
	$\tan\theta = \frac{2400 \pm \sqrt{2400^2 - 4(1125)(1253)}}{2(1125)}$	m1		
	$\tan\theta = 1.22\ ,\ 0.912$	A1F		PI
	$\theta\ =\ 50.7^\circ\ ,\ 42.4^\circ$	A1F	5	
(b)(ii)	$\theta = 42.4^\circ$	B1F		For the smaller angle
	$t = \frac{150}{40\cos\theta}$ and $\cos 42.4 > \cos 50.7$	E1	2	OE
			13	

Q	Solution	Marks	Total	Comments
4 (a)	$u_A = \dfrac{(-2\mathbf{i}+3\mathbf{j}+6\mathbf{k})140}{\sqrt{(2)^2+(3)^2+(6)^2}} = -40\mathbf{i}+60\mathbf{j}+120\mathbf{k}$	M1 A1		Simplification not needed
	$u_B = \dfrac{(2\mathbf{i}-\mathbf{j}+2\mathbf{k})60}{\sqrt{(2)^2+(1)^2+(2)^2}} = 40\mathbf{i}-20\mathbf{j}+40\mathbf{k}$	A1		Simplification not needed
	${}_Au_B = (-40\mathbf{i}+60\mathbf{j}+120\mathbf{k})-(40\mathbf{i}-20\mathbf{j}+40\mathbf{k})$ $= -80\mathbf{i}+80\mathbf{j}+80\mathbf{k}$	M1 A1F	5	Subtracting *B* from *A*
(b)	${}_Ar_B = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})-(-3\mathbf{i}+6\mathbf{j}+3\mathbf{k}) + t(-80\mathbf{i}+80\mathbf{j}+80k)$ **or** $(7\mathbf{i}-8\mathbf{j})+t(-80\mathbf{i}+80\mathbf{j}+80\mathbf{k})$	M1 A1F	2	A difference of initial p.v. $+t\times {}_Au_B$
(c)	${}_Ar_B = (7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}$	B1F		
	$s^2 = (7-80t)^2+(-8+80t)^2+(80t)^2$	B1F		
	$2s\dfrac{\mathrm{d}s}{\mathrm{d}t} = 2(7-80t)(-80)+2(-8+80t)(80)+2(80t)(80) = 0$	M1 A1F		Differentiation
	$240t = 15$	m1		Solving
	$t = 0.0625$ or $\dfrac{1}{16}$	A1F		
	$s^2 = (7-80\times 0.0625)^2+(-8+80\times 0.0625)^2+(80\times 0.0625)^2$	M1		
	$s = 6.16$ km or $\sqrt{38}$ km	A1F	8	
			15	
	<u>Alternative</u> (Not in the specification) *A* and *B* are closest $\Rightarrow$ ${}_A\mathrm{r}_B \cdot {}_A\mathrm{v}_B = 0$ $[(7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}]$ **.** $[-80\mathbf{i}+80\mathbf{j}+80\mathbf{k}]=0$ $-80(7-80t)+80(-8+80t)+80(80t)=0$ $240t=15$ $t=0.0625$	B1 M1 A1 A1 M1 A1		

Q	Solution	Marks	Total	Comments
5(a)	$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2(9.8)(2.5)$ $v = 7$	 M1 A1	 2	
(b)(i)	$\frac{w}{7} = e$ $w = 7e$ $0 = 7et - \frac{9.8}{2}t^2$ **or** **(** $0 = 7e - 9.8t$ **)** $t = \frac{10e}{7}$ **(** $t = 2 \times \frac{7e}{9.8}$ **)**	M1 M1 A1	 3	 Answer given
(ii)	$w' = 7e^2$ $0 = 7e^2 t' - \frac{9.8}{2}t'^2$ $t' = \frac{10e^2}{7}$	 B1	 1	 OE
(c)	$0^2 = (7e)^2 + 2(-9.8)h_2$ $h_2 = 2.5e^2$ $h_3 = 2.5e^2$ $0^2 = (7e^2)^2 + 2(-9.8)h_4$ $h_4 = 2.5e^4$ $h_5 = 2.5e^4$ Total distance $= 2.5 + 2(2.5e^2) + 2(2.5e^4)$ $= 2.5 + 5e^2 + 5e^4$	M1 A1 A1 m1 A1	 5	Or for correct method to find h_4
	Alternative (not in the specification) K.E. after each bounce $= e^2 \times$ K.E. before the bounce P.E. at max. height after each bounce $=$ $e^2 \times$ P.E. at max. height before the bounce Height after first bounce $= 2.5e^2$ Height after second bounce $= 2.5e^4$ Total $= 2.5 + 2(2.5e^2 + 2(2.5e^4)$ $= 2.5 + 5e^2 + 5e^4$	 (M1) (A1) (A1) (m1) (A1)		
(d)	Motion in vertical line, No air resistance, No energy loss, Instantaneous bounce	B1	1	
			12	

Q	Solution	Marks	Total	Comments
6 (a)	Perpendicular to the plane:			
	$y = -\frac{1}{2}gt^2\cos 20 + ut\sin 30$	M1		
	$0 = -4.9t^2\cos 20 + ut\sin 30$	M1		
	$t = 0.108589568u$ or $\frac{2u\sin 30}{g\cos 20}$	A1		
	Parallel to the plane:			
	$x = -\frac{1}{2}gt^2\sin 20 + ut\cos 30$	M1		
	$200 = -4.9(0.108589568u)^2\sin 20 + u(0.108589568u)\cos 30$	m1		
	$u^2 = 2693$	A1F		
	$u = 51.9$ or 51.894	A1F	7	Do not accept $\sqrt{2693}$
(b)	$\dot{y} = -gt\cos 20 + u\sin 30 = 0$	M1		
	$t = 2.817899$ or 2.817580214 or $\frac{51.9\sin 30}{g\cos 20}$	A1F		Accept 3 significant fig.
	The greatest $\perp$ distance $=$ $-\frac{1}{2}9.8(2.817899)^2\cos 20 + 51.9(2.817899)\sin 30$ or $-\frac{1}{2}9.8(\frac{51.894\sin 30}{9.8\cos 20})^2\cos 20 + 51.9(\frac{51.894\sin 30}{9.8\cos 20})\sin 30$	m1		
	$= 36.5622$ m or 36.5538 $= 36.6$ 3sf	A1F	4	
			11	
6 (a)	**Alternative:**			
	$x = 200\cos 20$	B1		
	$y = 200\sin 30$	B1		
	$200\cos 20 = u\cos 50t$	M1		
	$t = \frac{292.4}{u}$	A1		
	$200\sin 30 = \frac{1}{2}(-9.8)(\frac{292.4}{u})^2 + u\sin 50(\frac{292.4}{u})$	M1		
	$u^2 = 2693$	A1		
	$u = 51.9$	A1		
(b)	**Alternative:**			
	$0 = (u\sin 30)^2 - 2g\cos 20.s$	M1		
	$s = \frac{(51.9\sin 30)^2}{2(9.8)\cos 20}$	m1A1		
	$s = 36.6$	A1		

Q	Solution	Marks	Total	Comments
7 (a)	Momentum of A is unchanged $\perp$ to the line of centres			
	$4mu\sin 30 = 4mv_A \sin\alpha$	M1		
	$v_A = \dfrac{u}{2\sin\alpha}$ ....(1)	A1		
	C.L.M.:			
	$4mu\cos 30 = 4mv_A\cos\alpha + 3mv_B$	M1A1		
	$2\sqrt{3}u = 4v_A\cos\alpha + 3v_B$ ....(2)	A1F		OE
	Restitution along the line of centres:			
	$\dfrac{v_B - v_A\cos\alpha}{u\cos 30} = \dfrac{5}{9}$	M1A1		
	$v_B = v_A\cos\alpha + \dfrac{5\sqrt{3}u}{18}$ ....(3)	B1		Or equivalent, could be in part (b)
	$2\sqrt{3}u = 4\dfrac{u}{2\sin\alpha}\cos\alpha + 3\dfrac{u}{2\sin\alpha}\cos\alpha + \dfrac{15\sqrt{3}u}{18}$	m1		Solving (1), (2) and (3) Dependent on three M1s
	$\dfrac{7\sqrt{3}}{6} = \dfrac{7}{2\tan\alpha}$			
	$\tan\alpha = \sqrt{3}$			
	$\alpha = 60^\circ$ or $\dfrac{\pi}{3}$	A1F	10	
(b)	Impulse on B = Change in momentum of B along the line of centres			
	$v_B = \dfrac{u}{2\sin 60}\cos 60 + \dfrac{5\sqrt{3}u}{18}$			
	$v_B = \dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}$ $\left(= \dfrac{4\sqrt{3}}{9}\right)$	M1		
	$\mathrm{I} = 3m\left(\dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}\right) - 3m(0)$	M1		
	$\mathrm{I} = \dfrac{4mu}{\sqrt{3}}$ or $2.31mu$	A1F	3	
			13	
	TOTAL		**75**	